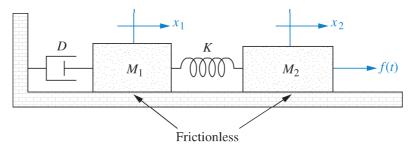
RME 3204: Control System Design

Assignment 02

Submission Date: 14 August 2025

P-1: Represent the system in state-space:



P-2: Represent the system in state-space & draw the corresponding block diagram:

$$\begin{array}{c|c}
R(s) & 24 & C(s) \\
\hline
s^3 + 9s^2 + 26s + 24 & \end{array}$$

P-3: Represent the system in state-space & draw the corresponding block diagram:

$$\begin{array}{c|c}
R(s) & s^2 + 7s + 2 \\
\hline
s^3 + 9s^2 + 26s + 24
\end{array}$$

P-4: Convert the state equations to transfer function.

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix} u$$
$$\mathbf{y} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \mathbf{x}$$

P-5: Solve the state-space equation to get y(t).

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -24 & -26 & -9 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} e^{-t}$$

$$y = [1 \quad 1 \quad 0] \mathbf{x}$$

$$\mathbf{x}(0) = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

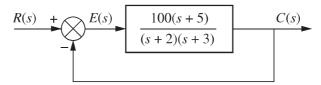
P-6: Solve the state-space equation to get y(t) where u(t) is a unit step.

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 \\ -8 & -6 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$
$$\mathbf{x}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

P-7: Obtain the response y(t) of the system where u(t) is the unit-step input occurring at t=0.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_1 \end{bmatrix} = \begin{bmatrix} -1 & -0.5 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0.5 \\ 0 \end{bmatrix} u, \quad \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

P-8: Find the state-space representation of feedback system.



P-9: Investigate the stability of the system.

$$\dot{\mathbf{x}} = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 7 & 1 \\ -3 & 4 & -5 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r$$

$$\mathbf{y} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \mathbf{x}$$

P-10: Investigate the controllability of the system.

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u = \begin{bmatrix} -1 & 1 & 2 \\ 0 & -1 & 5 \\ 0 & 3 & -4 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} u$$

P-11: Investigate the observability of the system.

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & -3 & -2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \mathbf{C}\mathbf{x} = [0 \quad 5 \quad 1]\mathbf{x}$$

P-12: Investigate the controllability & observability of the system.

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} r(t)$$
$$c(t) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}.$$