Engineering Economics

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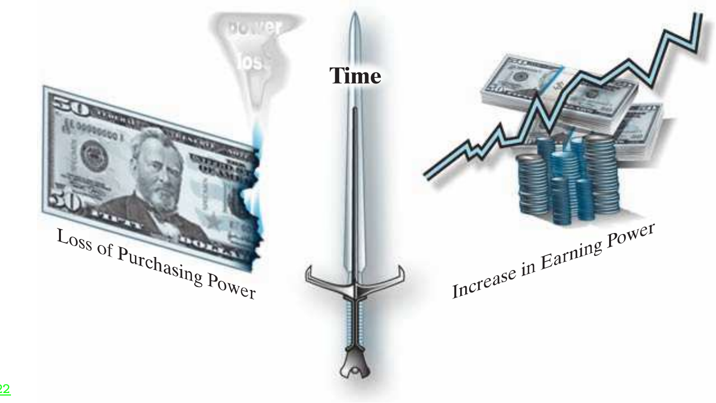
ME 307: Heat Transfer Equipment Design
http://zahurul.buet.ac.bd/ME307/

Time Value of Money



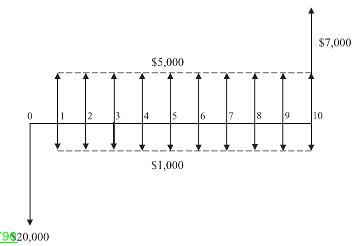
T1822

The time value of money. This is a two-edged sword whereby earning grows, but purchasing power decreases, as time goes by.

The Cash Flow Diagram

- Costs (or disbursements) are pointing down.
- Incomes are pointing up.
- Uniform yearly incomes and costs are indicated at the end of the year, even though they may be distributed throughout the year.

A heating system has an initial cost of \$20,000. The yearly operation and maintenance charges are \$1,000. Increased rent results in an extra \$5,000 per year of income. The heating plant has a life of 10 years, at which time it can be sold for \$7,000.



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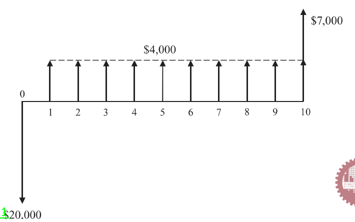

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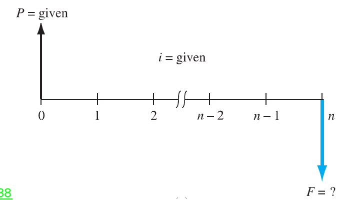
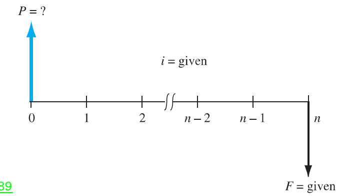
Single-payment formulas



T1788



T1789

$$\frac{F}{P} = (1+i)^n \equiv \left(\frac{F}{P}, i, n\right)$$

$$\frac{P}{F} = (1+i)^{-n} \equiv \left(\frac{P}{F}, i, n\right)$$

P	:	money considered at present time
F	:	money considered at future time
i	:	interest rate
n	:	number of time period considered
$\left(\frac{F}{P}, i, n\right)$	:	Single-payment compound amount factor
$\left(\frac{P}{F}, i, n\right)$	:	Single-payment present worth factor

Nominal and Effective Interest Rates

- Interest rates are often quoted as a nominal annual rate.
- If compounding factor is more frequent, the nominal rate does not represent true rate.
- If effective interest rate, i_{eff} for a nominal rate of i for m compounding periods in one year

$$i_{eff} = \left(\frac{F}{P}, \frac{i}{m}, m\right) - 1$$

- For example, the effective interest rate for an investment scenario of 6% compounded monthly is,

$$i_{eff} = \left(q + \frac{i}{12}\right)^{12} - 1 = \left(1 + \frac{0.06}{12}\right)^{12} - 1 = 0.0618 = 6.18\%$$

Example: ▷ An investment opportunity that provides an annual rate of return of 6% is available. Determine the future value of a $1,000 investment 10 years from now if the interest is compounded (a) annually and (b) monthly.

(a) annually compounded:

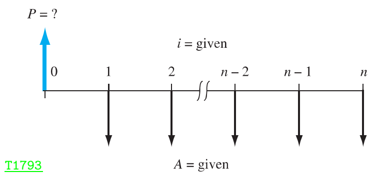
- $\left(\frac{F}{P}, i, n\right) = \left(\frac{F}{P}, 0.06, 10\right) = (1 + 0.06)^{10} = 1.79085$
- $F = P\left(\frac{F}{P}, i, n\right) = \$\ 1000(1.79085) = \$1790.85$

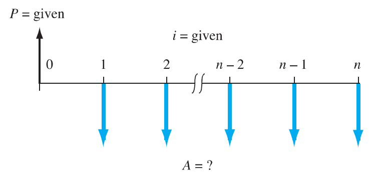
(b) monthly compounded: interest rate is divided equally over 12 months. So, i = 0.06/12 = 0.005, and n = (10)(12) = 120.

- $\left(\frac{F}{P}, i, n\right) = \left(\frac{F}{P}, 0.005, 120\right) = (1 + 0.005)^{120} = 1.81940$
- $F = P\left(\frac{F}{P}, i, n\right) = \$\ 1000(1.81940) = \$1819.40$

From an investment perspective, more frequent compounding results in a larger future sum because the interest is gaining interest.

Uniform-payment formulas

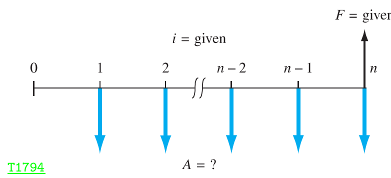


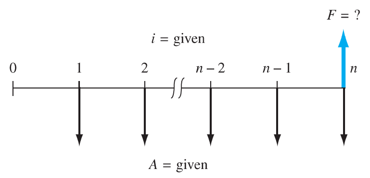


$$\frac{P}{A} = \frac{(1+i)^n - 1}{i(1+i)^n} \equiv \left(\frac{P}{A}, i, n\right)$$

$$\frac{A}{P} = \frac{i(1+i)^n}{(1+i)^n - 1} \equiv \left(\frac{A}{P}, i, n\right)$$

A : uniform periodic payments/incomes
$\left(\frac{P}{A}, i, n\right)$: uniform series present worth factor
$\left(\frac{A}{P}, i, n\right)$: capital recovery factor



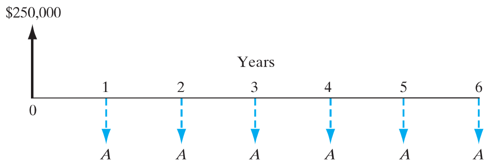


$$\frac{A}{F} = \frac{i}{(1+i)^n - 1} \equiv \left(\frac{A}{F}, i, n\right)$$

$$\frac{F}{A} = \frac{(1+i)^n - 1}{i} \equiv \left(\frac{F}{A}, i, n\right)$$

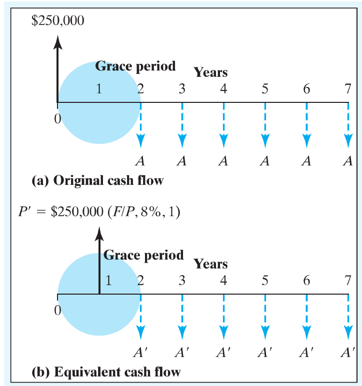
$\left(\frac{A}{F}, i, n\right)$: uniform series sinking fund factor
$\left(\frac{A}{P}, i, n\right)$: compound amount factor

Example: ▷ BioGen Company has borrowed $250,000 to purchase laboratory equipment for gene splicing. The loan carries an interest rate of 8% per year and is to be repaid in equal instalments over the next six years. Compute the amount of the annual instalment. [$54078.85]



T1797

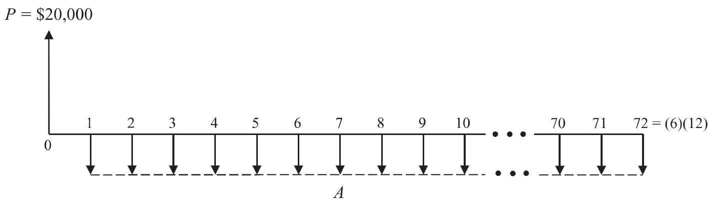
Example: ▷ BioGen Company has borrowed $250,000 to purchase laboratory equipment for gene splicing. BioGen wants to negotiate with the bank to defer the first loan repayment until the end of year 2 (but still desires to make six equal instalments at 8% interest). If the bank wishes to earn the same profit, what should be the annual instalment. [$58405.15]



T1823

Example: ▷ A bank is advertising new car loans at a nominal rate of 3.5%. Consider a situation where $20,000 is borrowed from this bank to buy a new car. The loan is to be paid back in equal monthly installments over a 6-year period. Determine the monthly payment and the effective interest rate of the loan.

[$308.37, 3.56%]



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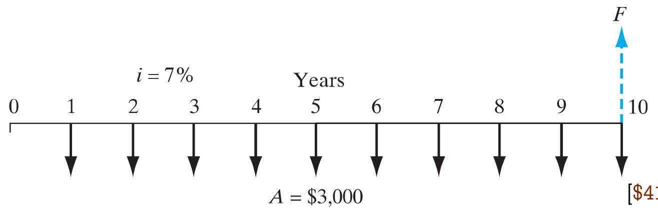
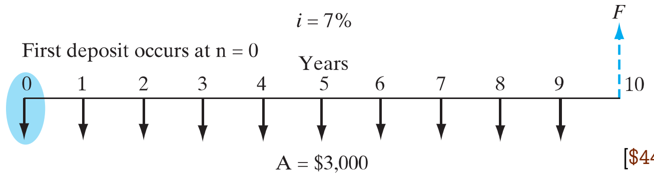
Example: ▷ Suppose you make an annual contribution of $3,000 to your savings account at the end of each year for 10 years. If the account earns 7% interest annually, how much can be withdrawn at the end of 10 years?



T1795

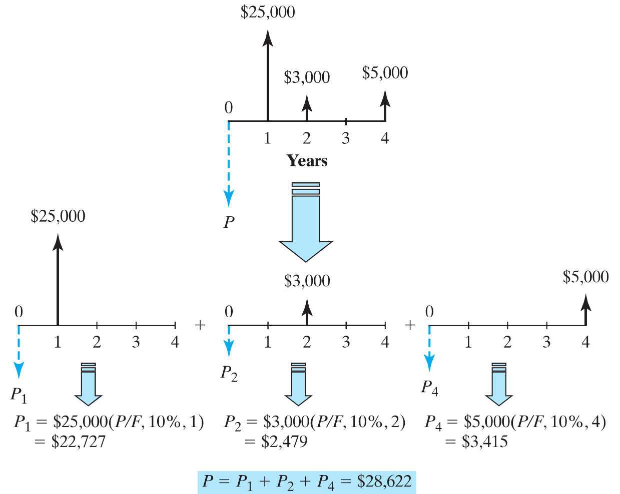
[$41449.34]

Example: ▷ In the previous example, the first deposit of the 10-deposit series was made at the end of period 1 and the remaining nine deposits were made at the end of each following period. Suppose that all deposits were made at the beginning of each period instead. How would you compute the balance at the end of period 10?



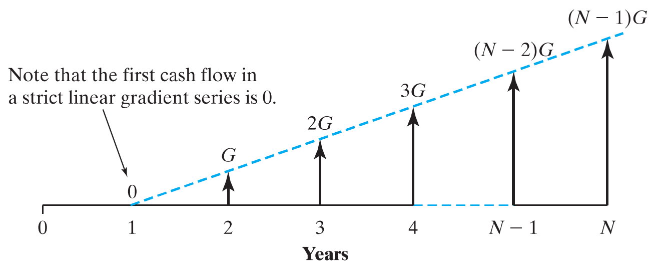
T1796 [$44350.80]

Example: ▷ Present Values of an Uneven Series by Decomposition into Single Payments



T1792

Linear-gradient Series Formula



T1798

$$\frac{P}{G} = \frac{(1+i)^n - 1}{i^2(1+i)^n} - \frac{n}{i(1+i)^n} \equiv \left(\frac{P}{G}, i, n\right)$$

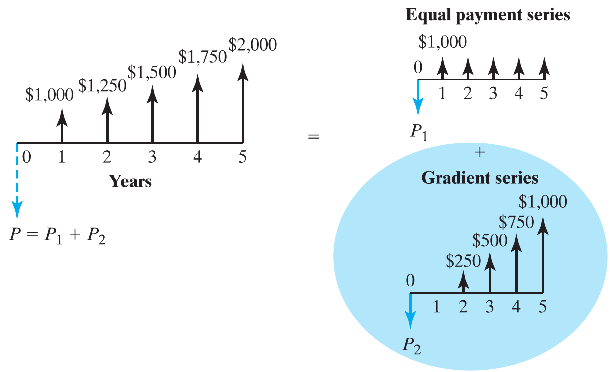
G : uniform cost/income gradient
$\left(\frac{P}{G}, i, n\right)$: gradient present worth facto

Interest Factors for Discrete Compounding

Name	Converts	Symbol	Computed by
Single Payment Compound Amount	P to F	$\left(\frac{F}{P}, i, n\right)$	$(1+i)^n$
Present Worth	F to P	$\left(\frac{P}{F}, i, n\right)$	$(1+i)^{-n}$
Uniform Series Sinking Fund	F to A	$\left(\frac{A}{F}, i, n\right)$	$\frac{i}{(1+i)^n - 1}$
Compound Amount	A to F	$\left(\frac{F}{A}, i, n\right)$	$\frac{(1+i)^n - 1}{i}$
Capital Recovery	P to A	$\left(\frac{A}{P}, i, n\right)$	$\frac{i(1+i)^n}{(1+i)^n - 1}$
Uniform Series Present Worth	A to P	$\left(\frac{P}{A}, i, n\right)$	$\frac{(1+i)^n - 1}{i(1+i)^n}$
Gradient Present Worth	G to P	$\left(\frac{P}{G}, i, n\right)$	$\frac{(1+i)^n - 1}{i^2(1+i)^n} - \frac{n}{i(1+i)^n}$

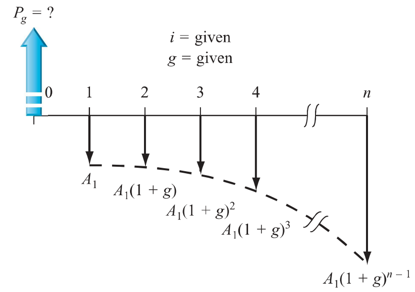
T1800

Example: ▷ Linear Gradient: Find P, Given G and N, $i = 12\%$. [$5204.03]



T1799

Geometric-gradient Series Formula

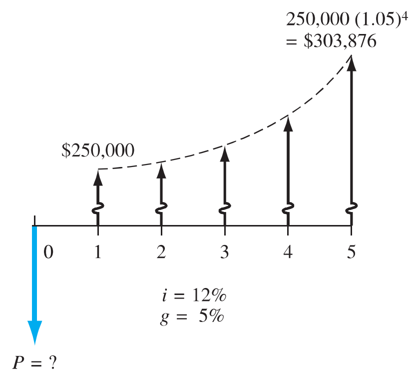


T1801

$$\frac{P_g}{A_1} = \begin{cases} \frac{1-\left(\frac{1+g}{1+i}\right)^n}{i-g} & \text{if } g \neq i \\ \frac{n}{1+i} & \text{if } g = i \end{cases}$$

g : geometric growth rate in decimal

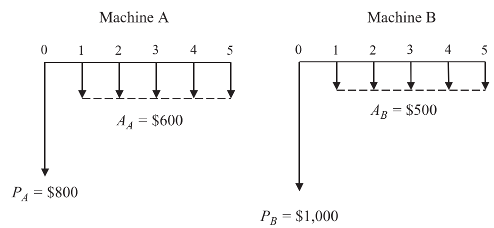
Example: ▷ A mechanical contractor has four employees whose combined salaries through the end of this year are $250,000. If he expects to give an average raise of 5% each year, calculate the present worth of the employees' salaries over the next 5 years. Let $i = 12\%$ per year. [$985012.74]



T1803

Present Worth Analysis

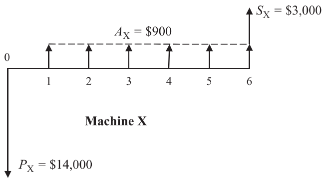
Example: ▷ Lives of 2 alternatives are equal: Two machines are being considered for a manufacturing process. Machine A has an initial cost of $800 with operating costs of $600 per year. Machine B costs $1,000 with operating costs of $500 per year. Both machines have a 5-year life with no salvage value. If MARR is 15%, compounded annually, which machine should be purchased?



T1824

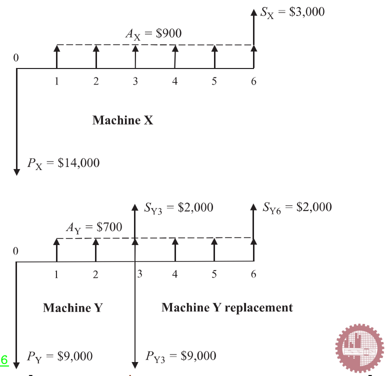
⇒ Select alternative with highest present worth (e.g., less -ve or more +ve).
[PW_A = - $2811, PW_B = - $2676. Select machine B.]

Example: ▷ Two alternatives: Machine X has an initial cost of \$14,000 with a salvage value after 6 years of \$3,000. Machine X results in an annual savings of \$900. Machine Y has an initial cost of \$9,000 and annual savings is \$700. Machine Y has a life of only 3 years, at which time it can be sold for \$2,000. The MARR for this project is 15%. Select suitable machine using a PW analysis.



T1825

[PW_X = -\$9297]

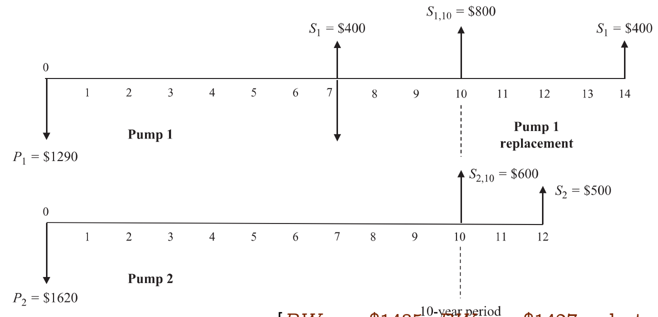


T1826

[PW_Y = -\$10089, Select machine X]

Example: ▷ Select a pump based on a 10-year service life if the MARR is 12%.
After 10 years, Pump 1 has salvage value of \$800 after 3 years, and pump 2 has salvage value \$600 after 10 years.

	Pump 1	Pump 2
Initial cost	\$1290	\$1620
Salvage value, at the end of useful life	\$400	\$500
Useful life	7 yrs	12 yrs



T1827

[PW_1 = -\$1435, PW_2 = -\$1427, select pump 2.]

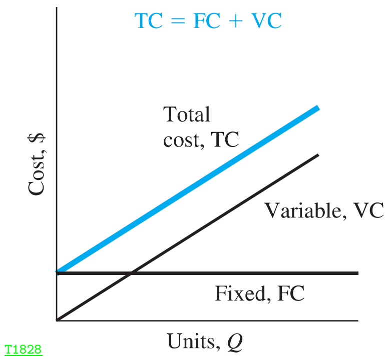
Annual Cost Analysis

⇒ The AC method can be used to compare alternatives with unequal economic lives. If the lives are different,the shorter-life alternative will be replaced with identical equipment (unless otherwise specified).

Example: ▷ Two machines are being considered for a manufacturing process. Machine A has an initial cost of \$800 with operating costs of \$600 per year. Machine B costs \$1,500 with operating costs of \$500 per year. Machine A has a 5-year life and Machine B has a 10-year life. Neither machine has a salvage value. If the MARR is 8%, which machine is the best economic alternative?

[AC_A = \$800, AC_B = \$724. Select machine B.]

Breakeven Analysis and Payback Period



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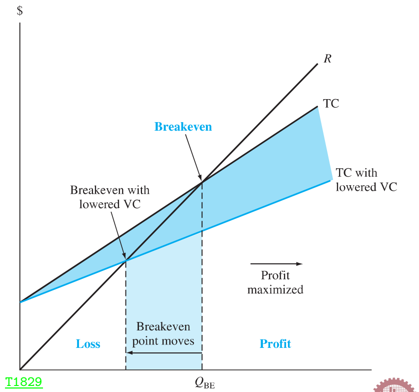


T1829

- r = revenue
- v = variable cost

$$Q_{BE} = \frac{FC}{r - v}$$

Example: ▷ Determine the total cost of a diesel generator operating over a 5-year period. Assume, the capital cost of the generator is $15,000, the annual output is 219 MWh and the maintenance costs are $500 per annum. The cost of producing each unit of electricity is $0.035/kWh.

Item	Type of cost	Calculation	Cost (£)
Capital cost of generator	Fixed	n.a.	15,000.00
Annual maintenance	Fixed	£500 × 5	2500.00
Fuel cost	Variable	219,000 × 0.035	7665.00
		Total cost =	25,165.00

T1830

Example: ▷ Determine the total cost of a diesel generator operating over a 5-year period. Assume the capital cost of the generator is $15000 and the maintenance costs are $500 per annum. The cost of producing electricity is $0.035/kWh. If electricity is bought from a local utility company costs an average of $0.061/kWh, determine the break-even point for the generator when:

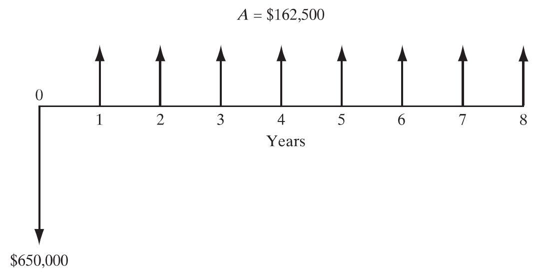
1. average output is 50 kW
2. average output is 70 kW

$$Q_{BE} = \frac{FC}{r - v}$$

- FC = $ (15000 + 500 x 5) = $17500
- r = $0.061/kWh x output
- v = $0.035/kWh x output
- $Q_{BE,1}$ = 17500/(0.061 x 50 - 0.035 x 50) = 13460 h
- $Q_{BE,2}$ = 17500/(0.061 x 70 - 0.035 x 70) = 9615 h

Payback Period

Example: ▷ Conventional Payback Period



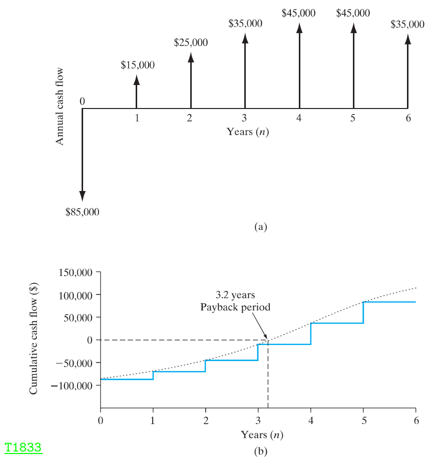
T1831

$$\text{Payback period} = \frac{\text{Initial cost}}{\text{Uniform annual benefit}} = \frac{\$650{,}000}{\$162{,}500} = 4 \text{ years.}$$

T1832

Payback Period with Salvage value

Example: ▷ Conventional Payback Period with salvage value.



Period	Cumulative cash flow
0	- $85000
1	- $70000
2	- $45000
3	- $10000
4	+ $45000
5	+ $80000
6	+ $115000

T1833