Thermodynamic Processes & Isentropic Efficiency

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ME 6101: Classical Thermodynamics

`http://zahurul.buet.ac.bd/ME6101/`

Steady-State, Steady Flow (SSSF) Processes

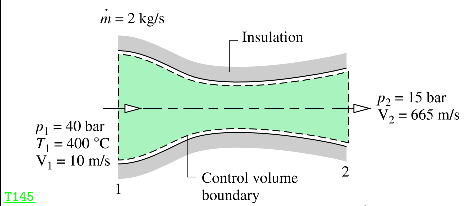
Assumptions:

- Control volume does not move relative to the coordinate frame.
- State of the mass at each point in the control volume does not vary with time.
- As for the mass that flows across the control surface, the mass flux and the state of this mass at each discrete area of flow on the control surface do not vary with time. The rates at which heat and work cross the control surface remain constant.

▷ Example: centrifugal compressor and turbines etc. at steady state.

Nozzles & Diffusers

Moran Ex. 4.3: ▷ Converging-diverging Steam Nozzle: Estimate A_2.



T145

⇒ Steady state ⇒ $dE_{cv}/dt = 0$

⇒ $Z_2 = Z_1$

⇒ $\dot{W}_{cv} = 0, \ \dot{Q} = 0$

⇒ $\dot{m}_i = \dot{m}_e = 2$ kg/s

$$\frac{dE_{cv}}{dt} = \dot{Q} - \dot{W}_{cv} + \sum_i \dot{m}_i \left(h_i + \frac{\mathbb{V}_i^2}{2} + gz_i \right) - \sum_e \dot{m}_e \left(h_e + \frac{\mathbb{V}_e^2}{2} + gz_e \right)$$

$$0 = (h_1 - h_2) + \tfrac{1}{2}(\mathbb{V}_1^2 - \mathbb{V}_2^2)$$

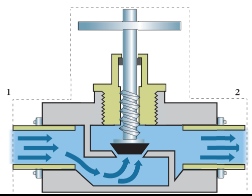
⇒ $h_2 = h_1 + \frac{1}{2}(\mathbb{V}_1^2 - \mathbb{V}_2^2)$

⇒ $\rho_2 = \rho(Steam, P = P_2, h = h_2) = 6.143$ kg/m^3

⇒ $\dot{m}_2 = \rho_2 A_2 \mathbb{V}_2 \Rightarrow A_2 = 4.896 \times 10^{-4}$ m^2 ◁

Throttling Devices

Cengel Ex. 5.8: ▷ R-134a enters the capillary tube as saturated liquid at 0.8 MPa and is throttled to 0.12 MPa. Determine x_2 and ΔT.



T148

⇒ Steady state ⇒ $dE_{cv}/dt = 0$

⇒ $Z_2 = Z_1$ & $\mathbb{V}_2 \simeq \mathbb{V}_1$

⇒ $\dot{Q} \simeq 0$ & $\dot{W}_{cv} = 0$

$$h_2 \cong h_1$$

⇒ $h_1 = enthalpy(R134a, P_1 = 0.8\ MPa, x_1 = 0.0)$

⇒ $h_1 = h_2 = h_{f,\ 0.12MPa} + x_2 h_{fg,\ 0.12MPa} \Rightarrow x_2 = 0.32$ ◁

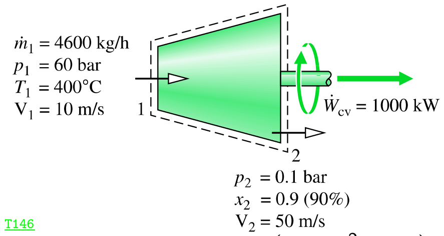
⇒ $T_1 = T(R134a, P_1 = 0.8\ MPa, x_1 = 0.0) \Rightarrow T_1 = 31.3^oC$

⇒ $T_2 = T(R134a, P_2 = 0.12\ MPa, sat.) \Rightarrow T_2 = -18.8^oC$

⇒ $\Delta T = -53.64^oC$ ◁

Turbines

Moran Ex 4.4: ▷ Heat Transfer from Steam Turbine: Determine heat loss.



T146

⇒ Steady state ⇒ $dE_{cv}/dt = 0$

⇒ $Z_2 = Z_1$

⇒ $\dot{m}_i = \dot{m}_e = (4600/3600)$ kg/s

$$\frac{dE_{cv}}{dt} = \dot{Q} - \dot{W}_{cv} + \sum_i \dot{m}_i \left(h_i + \frac{\mathbb{V}_i^2}{2} + gz_i\right) - \sum_e \dot{m}_e \left(h_e + \frac{\mathbb{V}_e^2}{2} + gz_e\right)$$

$$0 = \dot{Q} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \left(\frac{\mathbb{V}_1^2 - \mathbb{V}_2^2}{2}\right)\right]$$

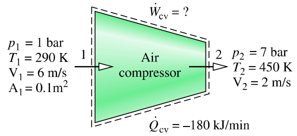
⇒ $h_1 = enthalpy(Steam, P = P_1, T = T_1)$

⇒ $h_2 = enthalpy(Steam, P = P_2, x = x_2)$

⇒ $\dot{Q}_{cv} = -63.61$ kW (heat loss) ◁

Compressors & Pumps

[Moran Ex. 4.5: ▷ Air Compressor Power: Determine power required, $\dot{W}_{cv}$.



T147

⇒ Steady state ⇒ $dE_{cv}/dt = 0$

⇒ $Z_2 = Z_1$

⇒ $\dot{Q}_{cv} = -180 kJ/min = -3.0$ kW

- $\dot{m} = \rho A \mathbb{V}$
- $\rho = P/RT$

$$\dot{W}_{cv} = \dot{Q} + \dot{m}\left[(h_1 - h_2) + \left(\frac{\mathbb{V}_1^2 - \mathbb{V}_2^2}{2}\right)\right]$$

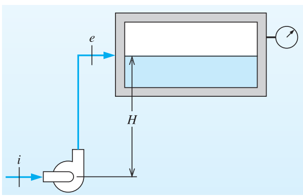
⇒ $h_1 - h_2 = C_p(T_1 - T_2) = -160.8$ kJ/kg

⇒ $\rho_1 = P_1/RT_1 = 1.20$ m³/kg

⇒ $\dot{m} = \rho_1 A_1 \mathbb{V}_1 = 0.721$ kg/s

⇒ $\dot{W}_{cv} = -119.4$ kW (work input required) ◁

Borgnakke Ex. 4.6: ▷ A water pump is located 15 m down in a well, taking water in at 10°C, 90 kPa at a rate of 1.5 kg/s. The exit line is a pipe of diameter 0.04 m that goes up to a receiver tank maintaining a gauge pressure of 400 kPa. Assume that the process is adiabatic, with the same inlet and exit velocities, and the water stays at 10°C. Find the required pump work.



T442

⇒ Steady state ⇒ $dE_{cv}/dt = 0$

⇒ $\dot{Q} = 0,\ V_i = V_e,\ T_i = T_e$

⇒ $P_i = 90$ kPa, $P_e = 501.325$ kPa.

⇒ $z_e - z_i = 15.0$ m

⇒ $\dot{m} = 1.5$ kg/s

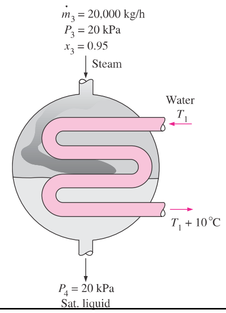
$$\frac{dE_{cv}}{dt} = \dot{Q} - \dot{W}_{cv} + \sum_i \dot{m}_i \left(h_i + \frac{\mathbb{V}_i^2}{2} + gz_i\right) - \sum_e \dot{m}_e \left(h_e + \frac{\mathbb{V}_e^2}{2} + gz_e\right)$$

$$\dot{W}_{cv} = \dot{m}\left[(h_1 - h_2) + \left(\frac{\mathbb{V}_1^2 - \mathbb{V}_2^2}{2}\right) + g(z_1 - z_2)\right]$$

$\dot{W}_{cv} = -0.822$ kW (work input required) ◁ : $\langle h_1 - h_2 = \frac{P_1 - P_2}{\rho}$, if ρ & T are constant. $\rangle$

Heat Exchangers

Cengel P5-85: ▷ Condenser Cooling Water: Determine $\dot{m}_{water}$.



T149

⇒ $\dot{W} = 0,\ \dot{Q} = 0,\ \Delta(KE) = 0,\ \Delta(PE) = 0$

⇒ $\dot{m}_3 = (20000/3600)$ kg/s

⇒ $\dot{m}_1 = \dot{m}_2$ & $\dot{m}_3 = \dot{m}_4$

⇒ $h_1 = h(H_2O,\ P = 100$ kPa, $T = 20)$

⇒ $h_2 = h(H_2O,\ P = 100$ kPa, $T = 30)$

⇒ $h_3 = h(H_2O,\ P = 20$ kPa, $x = 0.95)$

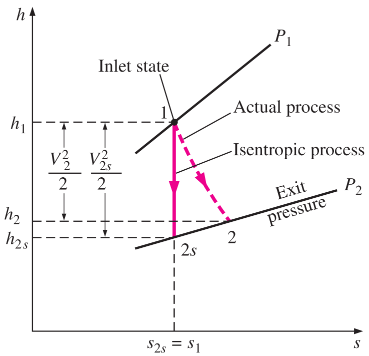
⇒ $h_4 = h(H_2O,\ P = 20$ kPa, $x = 0)$

$$0 = \sum(\dot{m}h)_i - \sum(\dot{m}h)_e$$

⇒ $(\dot{m}_3 h_3 + \dot{m}_1 h_1) = (\dot{m}_4 h_4 + \dot{m}_2 h_2)$

⇒ $m_{water} = m_1 = 297.4$ kg/s ◁

Isentropic Nozzle Efficiency

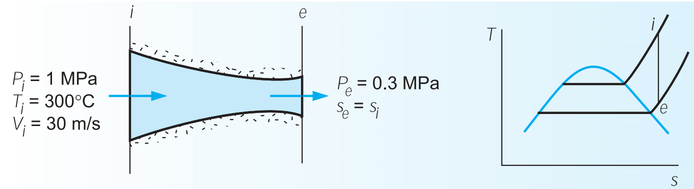


T181

- For nozzle: $W = 0,\ \Delta(PE) = 0,\ V_1 \sim 0 \Rightarrow h_1 = h_2 + \frac{V_2^2}{2}$

$\Rightarrow$ $$\eta_n \equiv \frac{\text{Actual KE at nozzle exit}}{\text{Isentropic KE at nozzle exit}} = \frac{V_2^2}{V_{2s}^2} \approx \frac{h_1 - h_2}{h_1 - h_{2s}}$$

Borgnakke Ex. 7.2: ▷ Isentropic steam flow through nozzle, $V_e = ?$



T185

- Continuity equation: $\dot{m}_i = \dot{m}_e = \dot{m}$
- First law: $0 = 0 - 0 + (h_i - h_e) + \frac{V_i^2 - V_e^2}{2} + 0$
- Second law: $s_i = s_e$.

$\Rightarrow$ $h_i = h(steam,\ P_i = 1\ MPa,\ T_i = 300^\circ C) = \checkmark$

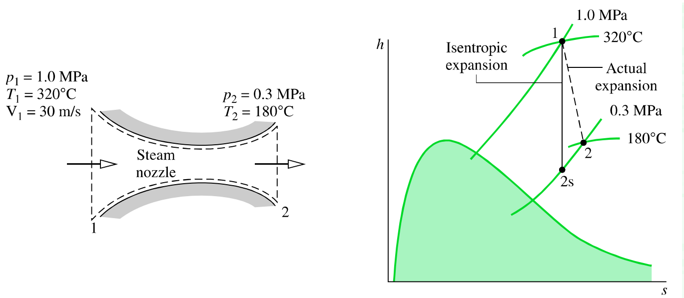
$\Rightarrow$ $s_i = s(steam,\ P_i = 1\ MPa,\ T_i = 300^\circ C) = \checkmark$

$\Rightarrow$ $s_e = s_i$ & $P_e = 0.3$ MPa: state 'e' defined.

$\Rightarrow$ $h_e = h(P_e = 0.3\ MPa,\ s_e = \checkmark) = \checkmark$

$\Rightarrow$ $\frac{V_e^2}{2} = (h_i - h_e) + \frac{V_i^2}{2} \Rightarrow V_e = 736.7$ m/s◁

Moran Ex. 6.13: ▷ Estimate nozzle efficiency.



T1083

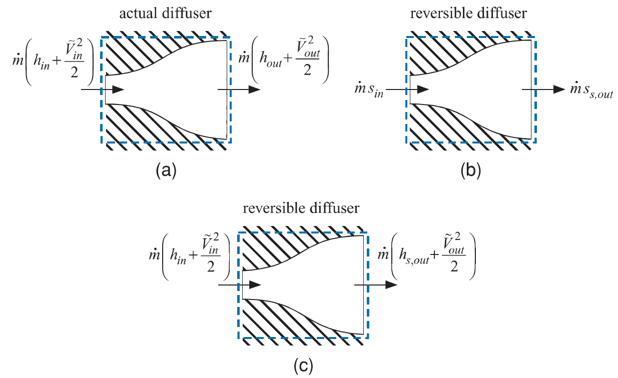
- Continuity equation: $\dot{m}_i = \dot{m}_e = \dot{m}$
- First law: $0 = 0 - 0 + (h_i - h_e) + \frac{V_i^2 - V_e^2}{2} + 0$, **valid for isentropic and actual cases.**

$\Rightarrow$ $\frac{V_e^2}{2} = (h_i - h_e) + \frac{V_i^2}{2} \Rightarrow V_e = \surd$

- For isentropic case, $V_{e,s} = \surd$

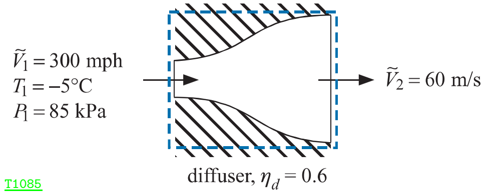
$\Rightarrow$ $\eta_N = \surd$

Diffuser Isentropic Efficiency



T1084

- Diffuser isentropic efficiency, $\eta_d \equiv \frac{P_{out} - P_{in}}{P_{s,out} - P_{in}}$
- Coefficient of pressure recovery, $K_p \equiv \frac{P_{out} - P_{in}}{\left(\frac{\rho_{in} V_{in}^2}{2}\right)}$

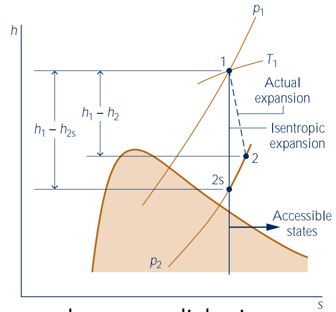


1. $P_2 = ?$
2. $\dot{\sigma}_{cv} = ?$
3. Area ratio, $A_2/A_1 = ?$
4. $K_p = ?$

T1085

- SSSF, $\dot{Q} = 0$, $\dot{W}_{cv} = 0$, $s_{s,2} = s_1$: reversible diffuser
- $0 = 0 - 0 + (h_i - h_e) + \frac{V_i^2 - V_e^2}{2} + 0 \rightarrow h_{s,2} = h_1 + \frac{V_1^2 - V_2^2}{2}$: s = case
- State $[s, 2]$ defined: $\rightarrow P_2 = P_1 + \eta_d(P_{s,2} - P_1) = \surd$ (89.9 kPa)
- First law (actual case: $h_2 = h_1 + \frac{V_1^2 - V_2^2}{2} \Rightarrow$ state [2] defined.
- $\dot{\sigma}_{cv} = s_2 - s_1 = \surd$ (10.3 J/kgK)
- $\dot{m} = \frac{V_1 A_1}{v_1} = \frac{V_2 A_2}{v_2}$
- $AR = \frac{A_2}{A_1} = \frac{v_2 V_1}{v_1 V_2} = \surd$
- $K_p \equiv \frac{P_{out} - P_{in}}{\left(\frac{\rho_{in} V_{in}^2}{2}\right)} = \surd$ (0.496).

Isentropic Turbine Efficiency

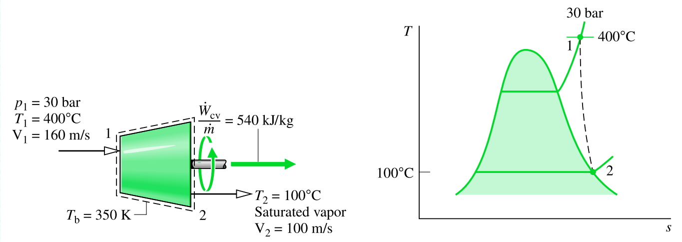


T176

- $\dot{w}_t = h_1 - h_2$: for steady-state, adiabatic expansion.
- $\frac{\dot{\sigma}_{cv}}{\dot{m}} = s_2 - s_1 \geqq 0$: states with $s_2 < s_1$ is not attainable with adiabatic expansion.
- $\dot{w}_t|_s = h_1 - h_{2s}$: state '2s' is for internally reversible expansion.

$\Rightarrow$ Isentropic turbine efficiency, $\boxed{\eta_t \equiv \frac{\dot{w}_t}{\dot{w}_t|_s} = \frac{h_1 - h_2}{h_1 - h_{2s}}}$

Moran Ex. 6.6: ▷ Entropy production in a steam turbine



T178

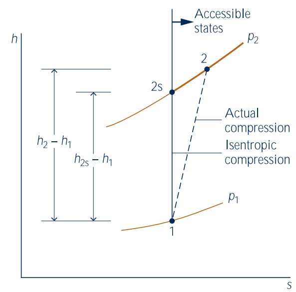
- Continuity equation: $\dot{m}_i = \dot{m}_e = \dot{m}$
- First law: $0 = q - w + (h_i - h_e) + \frac{V_i^2 - V_e^2}{2} + 0$
- Second law: $(s_e - s_i) = \frac{\dot{q}}{T_b} + \frac{\dot{\sigma}_{cv}}{\dot{m}}$.

$\Rightarrow$ $P_1 = 30$ bar, $T_1 = 400^oC \Rrightarrow h_1 = \checkmark$, $s_1 = \checkmark$

$\Rightarrow$ $T_2 = 100^oC$, $x_2 = 1.0 \Rrightarrow h_2 = \checkmark$, $s_2 = \checkmark$

$\Rightarrow$ $q = -23.92$ kJ/kg$\Rrightarrow \frac{\dot{\sigma}_{cv}}{\dot{m}} = 0.499$ kJ/kg.K ◁.

Isentropic Compressor and Pump Efficiencies

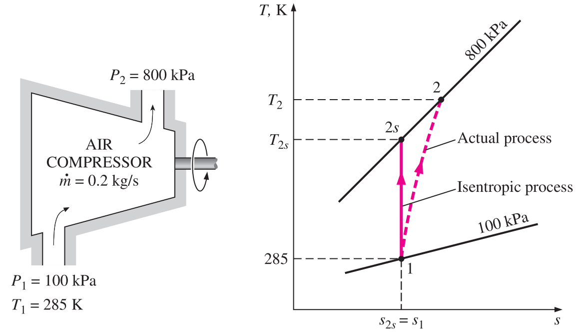


T177

- $-\dot{w}_c = (h_2 - h_1)$: for steady-state, adiabatic compression.
- $\frac{\dot{\sigma}_{cv}}{\dot{m}} = s_2 - s_1 \geqq 0$: states with $s_2 < s_1$ is not attainable with adiabatic compression.
- $-\dot{w}_c|_s = (h_{2s} - h_1)$

$\Rightarrow$ Isentropic compressor/pump efficiency, $\boxed{\eta_c \equiv \frac{-\dot{w}_c|_s}{-\dot{w}_c} = \frac{h_{2s} - h_1}{h_2 - h_1}}$

Cengel Ex. 7.15: ▷ Effect of efficiency on compressor power, if $\eta_c = 80\%$



T180

- $T_{2s} = T_1(P_2/P_1)^{(k-1)/k} = 516.5$ K
- $\eta_c = \frac{h_{2s}-h_1}{h_2-h_1} = \frac{c_P(T_{2s}-T_1)}{c_P(T_2-T_1)} \Rightarrow T_2 = 574.8$ K

$\Rightarrow -W_c|_s = \dot{m}(h_{2s} - h_1) = \dot{m}c_P(T_{2s} - T_1) = 46.5$ kW ◁

$\Rightarrow W_c = -\dot{m}(h_2 - h_1) = -\dot{m}c_P(T_2 - T_1) = -58.1$ kW ◁

Steady-state Flow Process

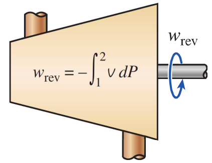
- First Law for SSSF: CV system & reversible process:

$\Rightarrow 0 = q_{12} - w_{12} + (h_1 - h_2) - \Delta(ke) - \Delta(pe)$

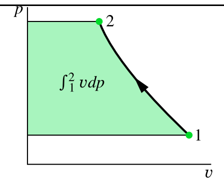
- 2^{nd} Tds Equation: $Tds = dh - vdP$
- $q_{12} = \int_1^2 Tds = (h_2 - h_1) - \int_1^2 vdP$
- $w_{12} = (h_2 - h_1) - \int_1^2 vdP + (h_1 - h_2) - \Delta(ke) - \Delta(pe)$

$\Rightarrow w_{12} = -\int_1^2 vdP - \Delta(ke) - \Delta(pe) \approx -\int_1^2 vdP$

$$w_{sf} = w_{12} = \int_1^2 vdP$$

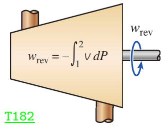


T182



T183

$$Pv^n = \text{constant},\ w_{sf} = w_{12} = -\int_1^2 vdP$$



T182

$$w_{12} : \begin{cases} = -\frac{n}{n-1}(P_2v_2 - P_1v_1) & = -\frac{nR(T_2-T_1)}{n-1} & : n \neq 1 \\ = -RT\ln\left(\frac{v_1}{v_2}\right) & = -RT\ln\left(\frac{P_2}{P_1}\right) & : n = 1 \end{cases}$$

$$q_{12} = h_2 - h_1 + w_{12}$$

Example: ▷ A compressor operates at steady state with nitrogen entering at 100 kPa and 20°C and leaving at 500 kPa. During this compression process, the relation between pressure and volume is $Pv^{1.3}$ =constant.

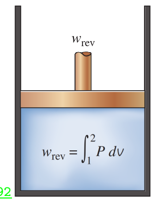
- $R = (8.314/28) = 0.2969$ kJ/kg.K, $c_P = 1.0$ kJ/kg.K

$\Rightarrow T_2/T_1 = (P_2/P_1)^{(n-1)/n} \Rightarrow T_2 = 425$ K.

$\Rightarrow w_{12} = -\frac{nR(T_2-T_1)}{n-1} = -169.5$ kJ/kg ◁

$\Rightarrow q_{12} = c_P(T_2 - T_1) + w_{12} = -37.5$ kJ/kg ◁

$$Pv^n = \text{constant},\ w_b = w_{12} = \int_1^2 Pdv$$



T192

$$w_{12} : \begin{cases} = -\frac{P_2v_2-P_1v_1}{n-1} & = -\frac{R(T_2-T_1)}{n-1} & : n \neq 1 \\ = -RT\ln\left(\frac{v_1}{v_2}\right) & = -RT\ln\left(\frac{P_2}{P_1}\right) & : n = 1 \end{cases}$$

$$q_{12} = u_2 - u_1 + w_{12}$$

Example: ▷ In a reversible process, nitrogen is compressed in a cylinder from 100 kPa and 20°C to 500 kPa. During this compression process, the relation between pressure and volume is $Pv^{1.3}$ =constant.

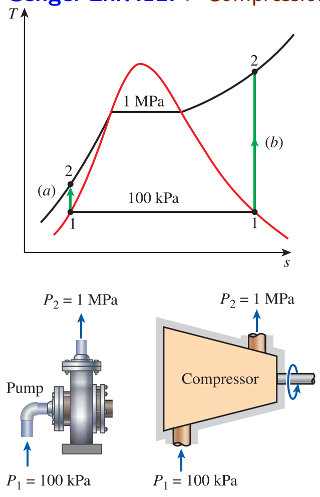
- $R = (8.314/28) = 0.2969$ kJ/kg.K, $c_v = R/(1.4-1) = 0.742$ kJ/kg.K

$\Rightarrow T_2/T_1 = (P_2/P_1)^{(n-1)/n} \Rightarrow T_2 = 425$ K.

$\Rightarrow w_{12} = -\frac{R(T_2-T_1)}{n-1} = -130.4$ kJ/kg ◁

$\Rightarrow q_{12} = c_V(T_2 - T_1) + w_{12} = -32.2$ kJ/kg ◁

Cengel Ex.7.12: ▷ Compression & Pumping Works



(a) Compressing a liquid (b) Compressing a vapor

T191

⇒ $w_P = -\int_1^2 v dP \simeq -v_f(P_2 - P_1) = -0.939 \text{ kJ/kg} \lhd$ as $v_f \simeq$ constant

⇒ $w_C = -(h_2 - h_1) = -518.6 \text{ kJ/kg} \lhd$

⇒ $|w_c| >> |w_p|$