

Second Law of Thermodynamics & Entropy

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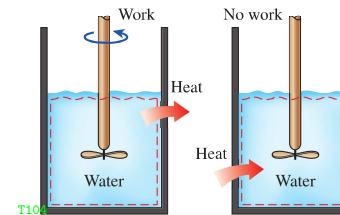
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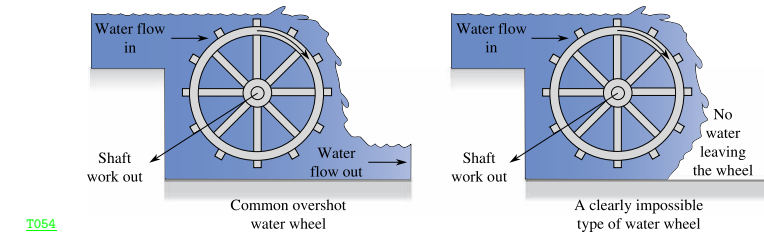
ME 6101: Classical Thermodynamics
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Some Observations in Work & Heat Conversions



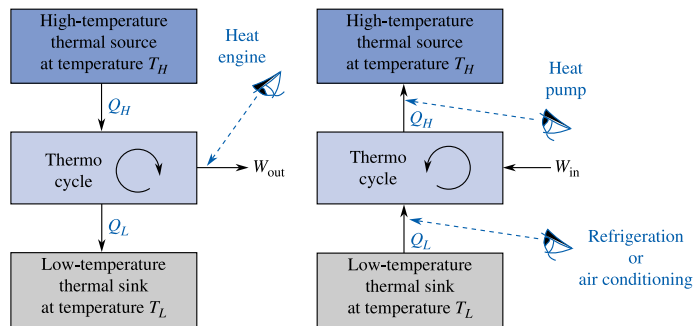
Work can always be converted to heat directly and completely, but the reverse is not true.



T054



Heat Engine: Classifications

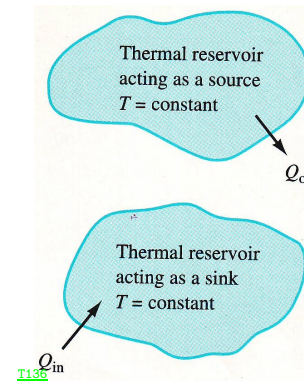


T058

- **Engine:** Thermal Efficiency, $\eta_{th} = \frac{W_{net,out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{Q_L}{Q_H}$
- **Refrigeration:** Coefficient of Performance, $COP_R = \frac{\text{Desired Output}}{\text{Required Input}} = \frac{Q_L}{W_{net,in}}$
- **Heat Pump:** Coefficient of Performance, $COP_{HP} = \frac{\text{Desired Output}}{\text{Required Input}} = \frac{Q_H}{W_{net,in}}$



Thermal Reservoir



T136

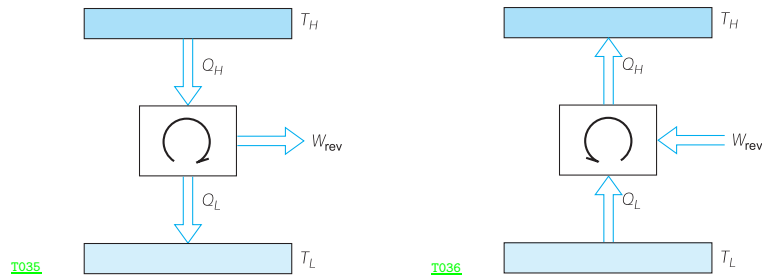
A **thermal reservoir** is a closed system with the following characteristics:

- Temperature remains uniform and constant during a process.
- Changes within the thermal reservoir are internally reversible.
- Heat transfer to or from a thermal reservoir only results in an increase or decrease in the internal energy of the reservoir.

A thermal reservoir is an idealization which in practice can be closely approximated. Large bodies of water, such as oceans and lakes, and the atmosphere behave essentially as thermal reservoirs.



Reversible Engines

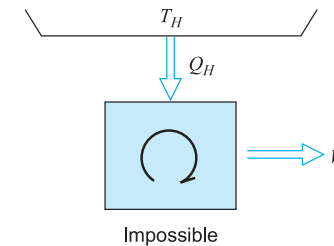


- A **reversible process** for a system is defined as a process that once having taken place can be reversed and in so doing leave no change in either the system or the surrounding.
- A reversible power cycle can be changed to a reversible refrigeration cycle by just reversing all the heat and work flow quantities.

Kelvin-Planck (KP) Statement

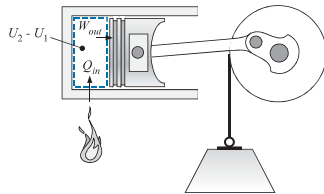
Kelvin-Planck (KP) statement

It is impossible to construct a device that will operate in a cycle and produce no effect other than the raising of a weight and the exchange of heat with a single reservoir.



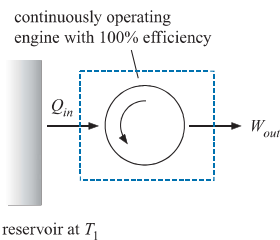
T028

$$W_{net} \leq 0 \quad \text{for single reservoir}$$

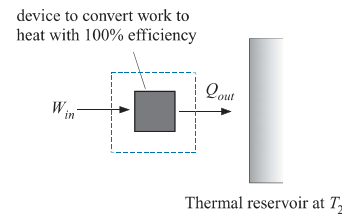


T755

A non-continuous process that converts heat to work with 100% efficiency.



T756

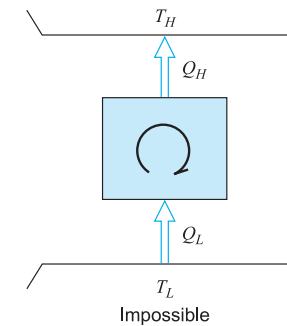


Process (a) violates the Second Law of Thermodynamics.

Clausius Statement

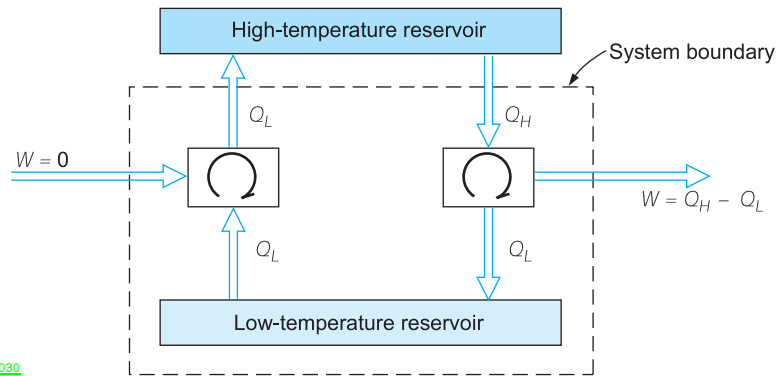
Clausius statement

It is impossible to construct a device that operates in a cycle and produces no effect other than the transfer of heat from a cooler body to a hotter body.



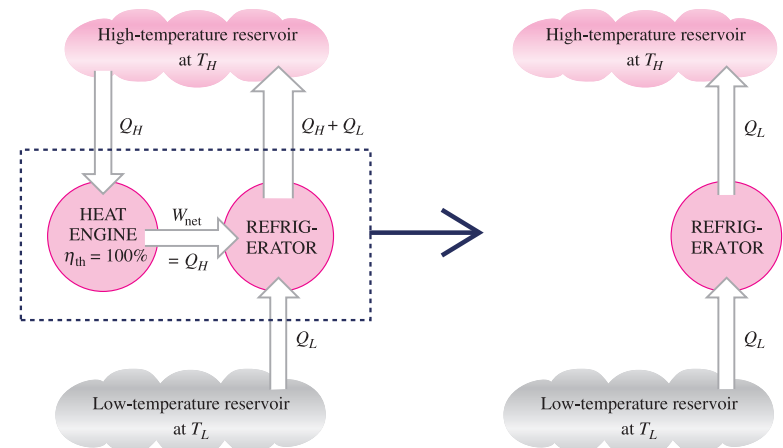
T029

Equivalence of Statements



T030

Violation of Clausius (C) statement \Rightarrow violation of Kelvin-Planck (KP) statement



T113

(a) A refrigerator that is powered by a 100 percent efficient heat engine

(b) The equivalent refrigerator

Violation of Kelvin-Planck (KP) statement \Rightarrow violation of Clausius (C) statement



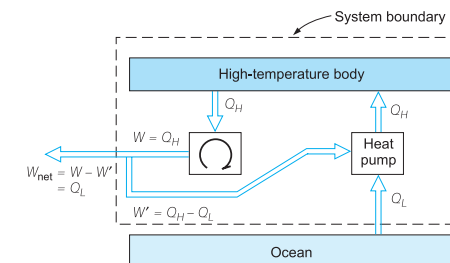
3 Observations of Two Statements

- 1 **Both are negative statements**; negative statements are impossible to prove directly. Every relevant experiment that has been conducted, either directly or indirectly, verifies the second law, and no experiment has ever been conducted that contradicts the second law. The basis of the second law is therefore experimental evidence.
- 2 **Both statements are equivalent**. Two statements are equivalent if the truth of either statement implies the truth of the other or if the violation of either statement implies the violation of the other.
- 3 **Both statements state the impossibility of Perpetual Motion Machine of 2nd Kind (PMM2)**.



Perpetual Motion Machines

- 1 A perpetual-motion machine of the first kind (PMM1) would create work from nothing or create energy, thus violating the first law.
- 2 A perpetual-motion machine of the second kind (PMM2) would extract heat from a source and then convert this heat completely into other forms of energy, thus violating the second law.



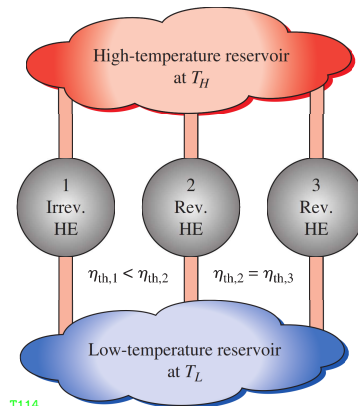
T140

A perpetual-motion machine of the second kind.

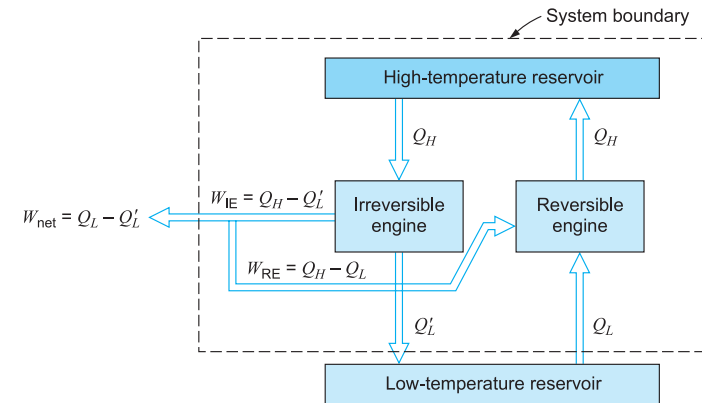


Carnot's Principles

- It is impossible to construct an engine that operates between two given reservoirs and is more efficient than a reversible engine operating between the same two reservoirs.
- All engines that operate on the Carnot cycle between two given constant-temperature reservoirs have the same efficiency.
- An absolute temperature scale may be defined which is independent of the measuring substances.



T114



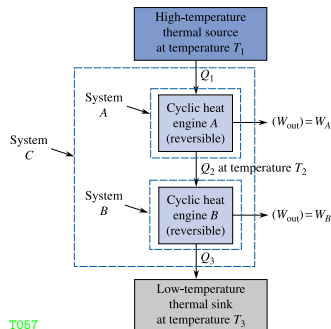
T032

If $\eta_{irr} > \eta_{rev} \Rightarrow |W_{IE}| > |W_{RE}|$ for same Q_H . Hence, composite system produces net work output while exchanging heat with a single reservoir \Rightarrow violation of K-P statement.

♣ Using same argument, $\eta_{rev} =$ same for same T_H & T_L

Thermodynamic Temperature Scale

Thermal efficiency of a reversible heat engine at a given set of reservoirs is independent of construction, design and working fluid of the engine.



T057

- $\eta_{th} = 1 - \frac{Q_L}{Q_H} = 1 - \psi(T_L, T_H)$
- $\frac{Q_1}{Q_2} = \psi(T_1, T_2), \frac{Q_2}{Q_3} = \psi(T_2, T_3)$
- $\frac{Q_1}{Q_3} = \psi(T_1, T_3) = \frac{Q_1}{Q_2} \cdot \frac{Q_2}{Q_3}$
- $\psi(T_1, T_3) = \underbrace{\psi(T_1, T_2) \cdot \psi(T_2, T_3)}_{\text{Not a function of } T_2}$

$$\Rightarrow \psi(T_1, T_2) = \frac{f(T_1)}{f(T_2)}, \psi(T_2, T_3) = \frac{f(T_2)}{f(T_3)}$$

$$\Rightarrow \psi(T_1, T_3) = \frac{f(T_1)}{f(T_3)} = \frac{f(T_1)}{f(T_2)} \cdot \frac{f(T_2)}{f(T_3)}$$

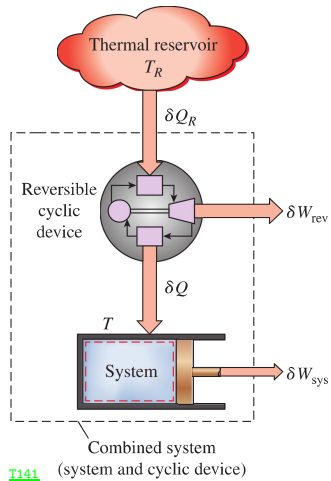
$$\Rightarrow \frac{Q_H}{Q_L} = \psi(T_H, T_L) = \frac{f(T_H)}{f(T_L)}$$

Kelvin proposed that, $f(T) = T \rightsquigarrow \frac{Q_H}{Q_L} = \frac{T_H}{T_L} \Rightarrow \eta_{rev. engine} = 1 - \frac{T_L}{T_H}$

Consequences of Second Law of Thermodynamics

- If a system is taken through a cycle and produces work, it must be exchanging heat with at least two reservoirs at 2 different temperatures.
- If a system is taken through a cycle while exchanging heat with a single reservoir, work must be zero or negative.
- Heat can never be converted continuously and completely into work, but work can always be converted continuously and completely into heat.
 - Work is a more valuable form of energy than heat.
 - For a cycle and single reservoir, $W_{net} \leq 0$.

Clausius Inequality



Clausius Inequality

$$\oint \frac{\delta Q}{T} \leq 0$$

$$\Rightarrow \delta W_{net} \equiv (\delta W_{rev} + \delta W_{sys}) = \delta Q_R - dU$$

$$\Rightarrow \frac{\delta Q_R}{\delta Q} = \frac{T_R}{T}$$

$$\Rightarrow \delta W_{net} = T_R \frac{\delta Q}{T} - dU$$

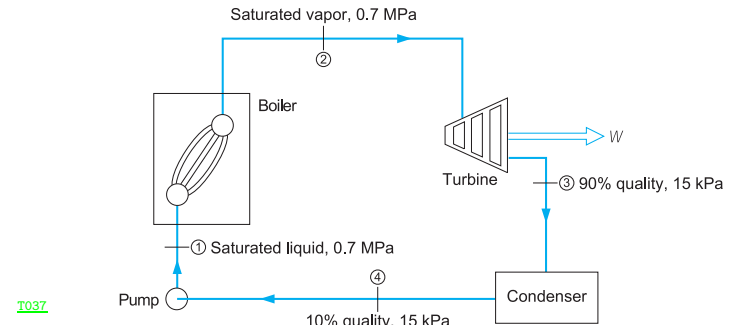
$$\Rightarrow W_{net} = T_R \int \frac{\delta Q}{T}$$

$$\Rightarrow \oint \frac{\delta Q}{T} \leq 0 \text{ as } W_{net} \leq 0$$

$$\oint \frac{\delta Q}{T} \begin{cases} = 0 & \text{reversible process} \\ < 0 & \text{irreversible process} \end{cases}$$

T141

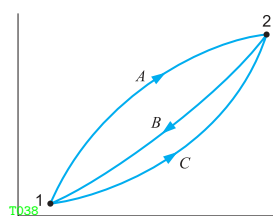
Example: Clausius Inequality: Steam Power Plant



T037

- At 0.7 MPa $T_{sat} = T_H = 164.95^\circ\text{C}$
- At 15 kPa $T_{sat} = T_L = 53.97^\circ\text{C}$
- $Q_H = Q_{12} = h_2 - h_1 = 2.066 \text{ MJ/kg}$
- $Q_L = Q_{34} = h_4 - h_3 = -1.898 \text{ MJ/kg}$
- $\Rightarrow \oint \frac{\delta Q}{T} = \frac{Q_H}{T_H} + \frac{Q_L}{T_L} = -1.086 \text{ kJ/kg} <$

Entropy (S): A Thermodynamic Property



For reversible process: $\oint \frac{\delta q}{T} = 0$

$$\Rightarrow \oint \frac{\delta q}{T} = \int_1^2 \left(\frac{\delta q}{T}\right)_A + \int_2^1 \left(\frac{\delta q}{T}\right)_B = 0 \quad \text{①}$$

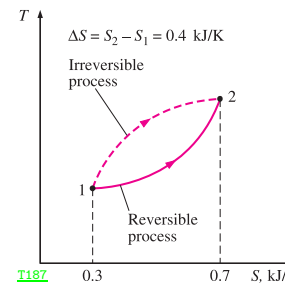
$$\Rightarrow \oint \frac{\delta q}{T} = \int_1^2 \left(\frac{\delta q}{T}\right)_C + \int_2^1 \left(\frac{\delta q}{T}\right)_B = 0 \quad \text{②}$$

$$\bullet \text{ ①} - \text{②} \Rightarrow \int_1^2 \left(\frac{\delta q}{T}\right)_A = \int_1^2 \left(\frac{\delta q}{T}\right)_C = \dots$$

Since $\int \delta q/T$ is same for all reversible processes/paths between state 1 & 2, this quantity is independent of path and is a function of end states only. This property is called **Entropy, S**.

$$ds \equiv \left(\frac{\delta q}{T}\right)_{rev} \Rightarrow \Delta s = s_2 - s_1 = \int_1^2 \left(\frac{\delta q}{T}\right)_{rev}$$

$$\delta q_{rev} = T ds$$



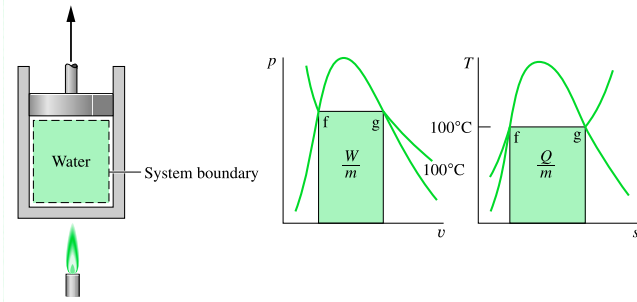
T187

- Entropy is a property, hence change in entropy between two end states is same for all processes, both reversible and irreversible.
- If no irreversibilities occur within the system boundaries of the system during the process, the system is internally reversible.
- For an internally reversible process, the change in the entropy is due solely for heat transfer. So, heat transfer across a boundary associated with it the transfer of entropy as well.

$$\int_1^2 \left(\frac{\delta q}{T}\right)_{rev} \equiv \text{Entropy transfer (or flux)}$$

- Entropy is transferred with heat, but there is no entropy transfer associated with energy transfer as work; work is entropy free.

Moran Ex. 6.1: Internally reversible heating in a piston-cylinder system.



T169

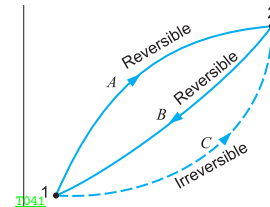
$$\Rightarrow w = \int_f^g P dv = P(v_g - v_f) = 101.325(1.673 - 0.001044) = 170 \text{ kJ/kg} <$$

$$\Rightarrow q = \int_f^g T ds = T(s_g - s_f) = T s_{fg} = 373.15 \cdot 6.0486 = 2256.8 \text{ kJ/kg} <$$

Also note that, $h_{fg} = 2257 \text{ kJ/kg} <$



Entropy Change for Irreversible CM Process



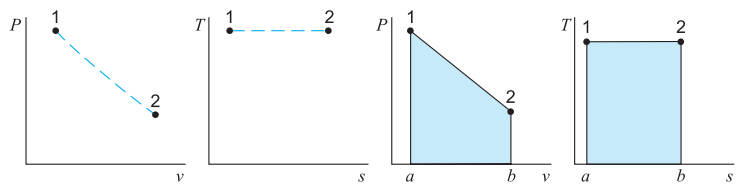
- For reversible process: $\oint \frac{\delta q}{T} = 0$
 $\Rightarrow \oint \frac{\delta q}{T} = \int_1^2 \left(\frac{\delta q}{T}\right)_A + \int_2^1 \left(\frac{\delta q}{T}\right)_B = 0$ ①
- For irreversible process: $\oint \frac{\delta q}{T} < 0$
 $\Rightarrow \oint \frac{\delta q}{T} = \int_1^2 \left(\frac{\delta q}{T}\right)_C + \int_2^1 \left(\frac{\delta q}{T}\right)_B < 0$ ②
- ① - ② $\Rightarrow \int_1^2 \left(\frac{\delta q}{T}\right)_A > \int_1^2 \left(\frac{\delta q}{T}\right)_C$

• Since path A is reversible, and since entropy is a property

$$\int_1^2 \left(\frac{\delta q}{T}\right)_A = \int_1^2 ds_A = \int_1^2 ds_C \Rightarrow \int_1^2 ds_C > \int_1^2 \left(\frac{\delta q}{T}\right)_C$$

• Since path C is arbitrary, for irreversible process

$$ds > \frac{\delta q}{T} \Rightarrow s_2 - s_1 > \int_1^2 \left(\frac{\delta q}{T}\right)_{irrev}$$



T170

Irreversible Process

Reversible Process

• For irreversible process:

$$w_{12} \neq \int_1^2 P dv \quad : \quad q_{12} \neq \int_1^2 T ds$$

So, the area underneath the path does not represent work and heat on the $P - v$ and $T - s$ diagrams, respectively.

• In irreversible processes, the exact states through which a system undergoes are not defined. So, irreversible processes are shown as dashed lines and reversible processes as solid lines.



Entropy Generation

$$dS \geq \frac{\delta Q}{T} \Rightarrow \delta \sigma \equiv dS - \frac{\delta Q}{T} \geq 0$$

$\sigma \triangleq$ Entropy produced (generated) by internal irreversibilities.

$$S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T}\right)_b + \sigma$$

entropy change	entropy transfer	entropy production
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$$\sigma : \begin{cases} > 0 & \text{irreversible process} \\ = 0 & \text{internally reversible process} \\ < 0 & \text{impossible process} \end{cases}$$

T173

• For CM system: $dS_{CM} = \frac{\delta Q}{T} + \delta \sigma$

• CM system, with heat transfer occurring at several boundaries, if T_i is the temperature at point where δQ_i takes place, then

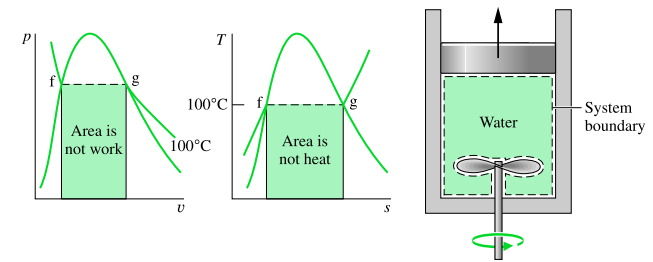
$$\frac{dS_{CM}}{dt} = \sum \frac{\delta \dot{Q}_i}{T_i} + \delta \dot{\sigma}$$



- Change in entropy of any CM system is due to only **2** physical effects:
 - Heat transfer to/from the system as measured by entropy transfer/flux, $\delta Q/T$.
 - Presence of irreversibilities within the system & its contribution is measured by entropy production, $\sigma \geq 0$.
- Only way to decrease the entropy of a closed system is to transfer of heat from it. In this case, heat transfer contribution must be more -ve than the +ve contribution of any internal irreversibility.
- Reversible process: $ds = \delta q/T$ & adiabatic process: $\delta q = 0$
 $\delta q/T = 0 \Rightarrow s = \text{constant}$: for reversible adiabatic process.
- All isentropic processes are not necessarily reversible & adiabatic. Entropy can remain constant during a process if heat removal balances the contribution due to irreversibility.



Moran Ex. 6.2: Irreversible process of water.



T172

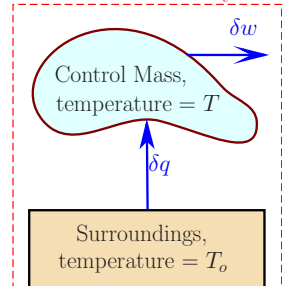
- $du + dke + dpe = \delta q - \delta w \Rightarrow du = -\delta w$
- $\Rightarrow \frac{W}{m} = -\int_f^g du = -(u_g - u_f) = -2087.56 \text{ kJ/kg} <$
 Note that, the work input by stirring is greater in magnitude than the work done by the water as it expands (170 kJ/kg).
- $\delta(\sigma/m) = ds - \frac{\delta q}{T} = ds - 0 = ds$
- $\Rightarrow \frac{\sigma}{m} = s_g - s_f = 6.048 \text{ kJ/kg.K} <$



Principle of Increase of Entropy

$$ds \geq \frac{\delta q}{T}$$

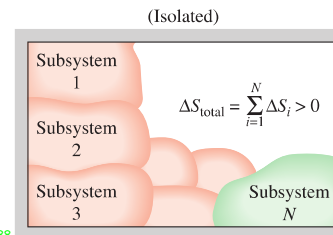
Adiabatic or isolated system



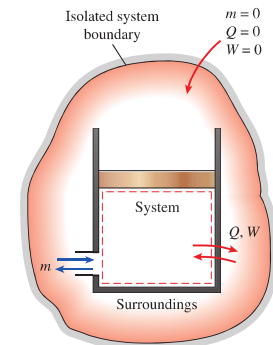
- For CM: $ds_{CM} \geq \frac{\delta q}{T}$.
- For surroundings, reversible heat transfer: $ds_{surr} = \frac{-\delta q}{T_o}$.
- $\Rightarrow ds_{net} = ds_{sys} + ds_{surr} \geq \delta q \left[\frac{1}{T} - \frac{1}{T_o} \right]$
- If $T > T_o \Rightarrow \delta q < 0 \Rightarrow ds_{net} \geq 0$
- If $T < T_o \Rightarrow \delta q > 0 \Rightarrow ds_{net} \geq 0$

$$ds_{net} \geq 0$$

Entropy change for an isolated system cannot be negative.



T188



T189

Entropy change of an isolated system is the sum of the entropy changes of its components, and is never less than zero. A system and its surroundings form an isolated system.

$$ds_{isolated} \geq 0$$



Example: ▷ Suppose that 1 kg of saturated water vapour at 100°C is to a saturated liquid at 100°C in a constant-pressure process by heat transfer to the surrounding air, which is at 25°C. What is the net increase in entropy of the water plus surroundings?

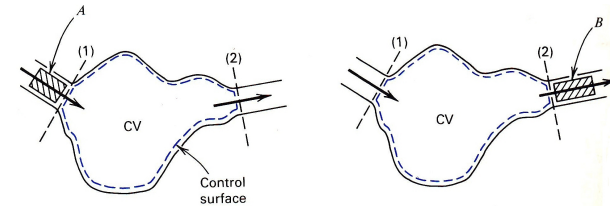
$$\Delta S_{net} = \Delta S_{sys} + \Delta S_{surr}$$

- $\Delta S_{sys} = -s_{fg} = -6.048 \text{ kJ/kg.K}$
 - $\Delta S_{surr} = \frac{q}{T_o} = \frac{h_{fg}}{T_o} = \frac{2257}{298} = 7.574 \text{ kJ/kg.K}$
- ⇒ $\Delta S_{net} = 1.533 \text{ kJ/kg.K} <$

So, increase in net entropy.



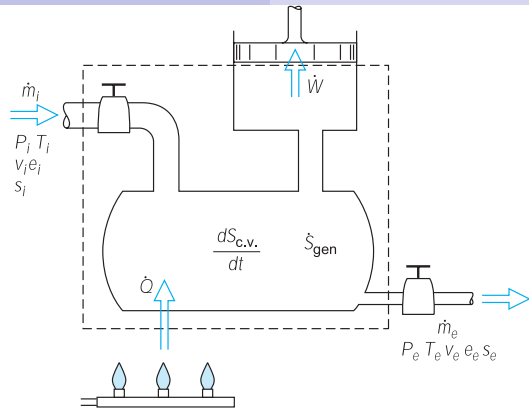
Second Law Analysis for CV System



T320
 at t at $t + \Delta t$
 CM (time t) : region A + CV CM ($t + \Delta t$) : CV + region B
 $S_{CM,t} = S_A + S_{CV,t}$ $S_{CM,t+\Delta t} = S_B + S_{CV,t+\Delta t}$

- $\frac{S_{CM,t+\Delta t} - S_{CM,t}}{\Delta t} = \frac{S_{CV,t+\Delta t} - S_{CV,t}}{\Delta t} + \frac{S_B - S_A}{\Delta t}$
- $\frac{dS_{CM}}{dt} = \frac{dS_{CV}}{dt} + \dot{m}_B s_B - \dot{m}_A s_A$
- $\frac{dS_{CM}}{dt} = \sum \frac{\dot{Q}_i}{T_i} + \dot{\sigma}$

$$\frac{dS_{CV}}{dt} = \sum \frac{\dot{Q}_i}{T_i} + \sum_i (\dot{m}s)_i - \sum_e (\dot{m}s)_e + \dot{\sigma}_{CV}$$



T042

$$\frac{dS_{cv}}{dt} = \sum_j \frac{\dot{Q}_j}{T_j} + \sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e + \dot{\sigma}_{cv}$$

rate of entropy change rates of entropy transfer rate of entropy production

T175



$$\frac{dS_{CV}}{dt} = \sum_j \frac{\dot{Q}_j}{T_j} + \sum (\dot{m}s)_i - \sum (\dot{m}s)_e + \dot{\sigma}_{CV}$$

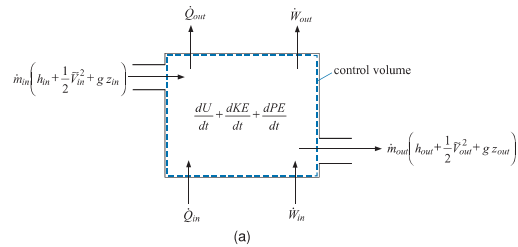
- For CM systems: $\dot{m}_i = 0, \dot{m}_e = 0 \Rightarrow \frac{dS_{CM}}{dt} = \sum_j \frac{\dot{Q}_j}{T_j} + \dot{\sigma}$
- For steady-state steady-flow (SSSF) process: $dS_{CV}/dt = 0$.
- For 1-inlet & 1-outlet SSSF process: $\dot{m}_i = \dot{m}_e$.

⇒ $(s_e - s_i) = \sum_j \frac{\dot{q}_j}{T_j} + \frac{\dot{\sigma}_{CV}}{\dot{m}}$

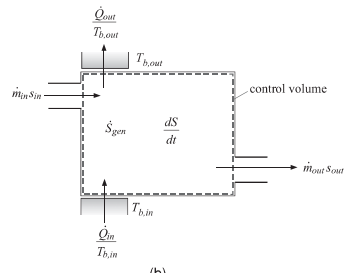
- For adiabatic 1-inlet & 1-outlet SSSF process:

⇒ $(s_e - s_i) = \frac{\dot{\sigma}_{CV}}{\dot{m}} \Rightarrow s_e \geq s_i$





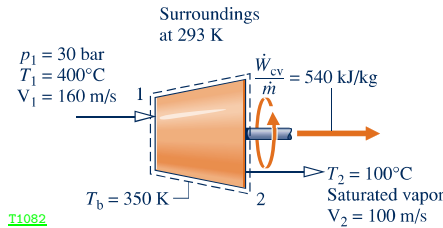
(a)



(a) Energy balance (b) Entropy balance.

T757

Moran Ex. 6-6 ▷ Determine the rate at which entropy is produced within the turbine per kg of steam flowing, in kJ/kgK.



T1082

- SSSF: $\frac{dS_{CV}}{dt} = 0, \dot{m}_i = \dot{m}_e = \dot{m}$
- $\sum \frac{\dot{Q}_i}{T_i} = \frac{\dot{Q}}{T_b}: T_b = 350 \text{ K.}$
- $z_i = z_e$
- States ① & ②: defined.

$$\bullet \frac{dE_{CV}}{dt} = \dot{Q} - \dot{W}_{cv} + \dot{m}_i \left(h_i + \frac{V_i^2}{2} + gz_i \right) - \dot{m}_e \left(h_e + \frac{V_e^2}{2} + gz_e \right)$$

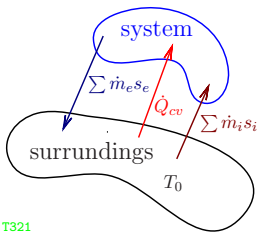
$$\Rightarrow \dot{Q} = \dot{W}_{cv}$$

$$\bullet \frac{dS_{CV}}{dt} = \sum \frac{\dot{Q}_i}{T_i} + (\dot{m}s)_i - (\dot{m}s)_e + \dot{\sigma}_{CV}$$

$$\Rightarrow \dot{\sigma}_{CV} = \dot{\sigma}$$



Principle of Increase of Entropy



$$\bullet \frac{dS_{CV}}{dt} = \sum \frac{\dot{Q}_i}{T_i} + \sum_i (\dot{m}s)_i - \sum_e (\dot{m}s)_e + \dot{\sigma}_{CV}$$

$$\Rightarrow \frac{dS_{CV}}{dt} = \sum \frac{\dot{Q}_i}{T_i} + \sum_i (\dot{m}s)_i - \sum_e (\dot{m}s)_e + \dot{\sigma}_{CV}$$

$$\bullet \frac{dS_{surr}}{dt} = \sum \frac{\dot{Q}_i}{T_i} + \sum_i (\dot{m}s)_i - \sum_e (\dot{m}s)_e + \dot{\sigma}_{surr}$$

$$\Rightarrow \frac{dS_{surr}}{dt} = -\frac{\dot{Q}_{CV}}{T_0} - \sum_i (\dot{m}s)_e + \sum_e (\dot{m}s)_e + \dot{\sigma}_{surr}$$

T321

$$\bullet \frac{dS_{net}}{dt} = \frac{dS_{CV}}{dt} + \frac{dS_{surr}}{dt} = \sum \frac{\dot{Q}_i}{T_i} - \frac{\dot{Q}_{CV}}{T_0} + \dot{\sigma}_{tot}$$

$$\bullet \dot{\sigma}_{tot} \geq 0, \text{ so}$$

$$\boxed{\frac{dS_{net}}{dt} = \frac{dS_{CV}}{dt} + \frac{dS_{surr}}{dt} \geq 0}$$



Entropy of Ideal Gas

- **First Law:** $\delta q - \delta w = du$
 - **Reversible process:** $\delta w = Pd v : \delta q = Tds$
 - **Ideal gas:** $Pv = RT : du = c_v dT : dh = c_p dT$
- $$\Rightarrow du = -Pd v + Tds \Rightarrow \boxed{Tds = du + Pd v} : 1^{st} \text{ Tds Equation.}$$
- $$\Rightarrow ds = c_v \frac{dT}{T} + \frac{P}{T} dv = c_v \frac{dT}{T} + R \frac{dv}{v} : \text{For ideal gas.}$$
- $$\Rightarrow \boxed{s(T_2, v_2) - s(T_1, v_1) = \int_{T_1}^{T_2} c_v \frac{dT}{T} + R \ln \left(\frac{v_2}{v_1} \right)}$$
- $$\Rightarrow \boxed{s_2 - s_1 = c_v \ln \left(\frac{T_2}{T_1} \right) + R \ln \left(\frac{v_2}{v_1} \right)} : \text{Ideal gas with } c_v = \text{constant.}$$
- $$\Rightarrow h = u + Pv \Rightarrow dh = du + Pd v + v dP = \delta q_{rev} + v dP = Tds + v dP$$
- $$\Rightarrow dh = Tds + v dP \Rightarrow \boxed{Tds = dh - v dP} : 2^{nd} \text{ Tds Equation.}$$
- $$\Rightarrow \boxed{s(T_2, P_2) - s(T_1, P_1) = \int_{T_1}^{T_2} c_p \frac{dT}{T} - R \ln \left(\frac{P_2}{P_1} \right)}$$
- $$\Rightarrow \boxed{s_2 - s_1 = c_p \ln \left(\frac{T_2}{T_1} \right) - R \ln \left(\frac{P_2}{P_1} \right)} : \text{Ideal gas with } c_p = \text{constant.}$$



Isentropic Process: $s = \text{constant} \Rightarrow \Delta s = 0$

- $s_2 - s_1 = c_p \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{v_2}{v_1}\right)$

$$\Rightarrow \ln\left(\frac{T_2}{T_1}\right) = -\frac{R}{c_v} \ln\left(\frac{v_2}{v_1}\right) = -(k-1) \ln\left(\frac{v_2}{v_1}\right) = \ln\left(\frac{v_1}{v_2}\right)^{(k-1)}$$

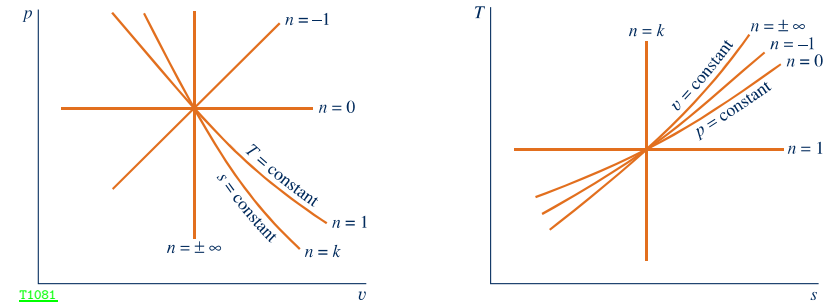
$$\Rightarrow \boxed{\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{(k-1)} \quad (\text{ideal gas, } s_1 = s_2, \text{ constant } k)}$$

- $s_2 - s_1 = c_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right)$

$$\Rightarrow \ln\left(\frac{T_2}{T_1}\right) = \frac{R}{c_p} \ln\left(\frac{P_2}{P_1}\right) = \frac{(k-1)}{k} \ln\left(\frac{P_2}{P_1}\right) = \ln\left(\frac{P_2}{P_1}\right)^{\frac{(k-1)}{k}}$$

$$\Rightarrow \boxed{\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{(k-1)}{k}} \quad (\text{ideal gas, } s_1 = s_2, \text{ constant } k)}$$

$$\Rightarrow \boxed{Pv^k = \text{constant} \quad (\text{ideal gas, } s_1 = s_2, \text{ constant } k)}$$



T1081

Polytropic processes on P-v and T-s diagrams



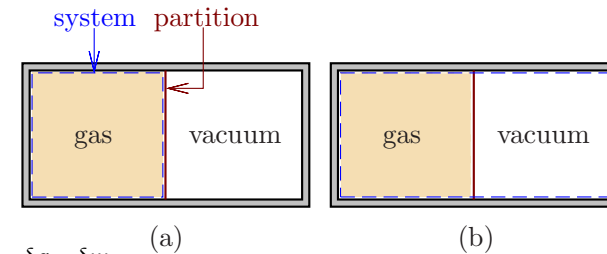
Example: ▷ Determine the change in specific entropy, in KJ/kg-K, of air as an ideal gas undergoing a process from 300 K, 1 bar to 400 K, 5 bar. Because of the relatively small temperature range, we assume a constant value of $c_p = 1.008$ KJ/kg-K.

$$\begin{aligned} \Delta s &= c_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right) \\ &= \left(1.008 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right) \ln\left(\frac{400\text{K}}{300\text{K}}\right) - \left(\frac{8.314 \text{ kJ}}{28.97 \text{ kg}\cdot\text{K}}\right) \ln\left(\frac{5\text{bar}}{1\text{bar}}\right) \\ &= -0.1719 \text{ kJ/kg}\cdot\text{K} < \end{aligned}$$

- Note that, for isentropic compression, $T_{2s} = T_1(P_2/P_1)^{(k-1)/k} = 475$ K. Hence, entropy change is (-) ve because of cooling of air from 475 K to 400 K.
- Comment on the results is final state is 5 bar and 500 K.



Example: ▷ Air is contained in one half of an insulated tank. The other side is completely evacuated. The membrane is punctured and air quickly fills the entire volume. Calculate the specific entropy change of the isolated system.



T022

- $du = \delta q - \delta w$
- $s_2 - s_1 = c_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{v_2}{v_1}\right)$
- $\Rightarrow w = 0, q = 0 \Rightarrow du = 0 \Rightarrow c_v dT = 0 \Rightarrow T_2 = T_1.$
- $\Rightarrow s_2 - s_1 = c_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{v_2}{v_1}\right) = 0 + 287 \ln(2) = 198.93 \text{ kJ/kg}\cdot\text{K} <$
- Note that: $s_2 - s_1 = 198.93 \text{ kJ/kg}\cdot\text{K} > \underbrace{\left(\frac{\delta q}{T}\right)}_{=0} \Rightarrow \Delta s > 0$

