Irreversibility: Work & Heat Transfer

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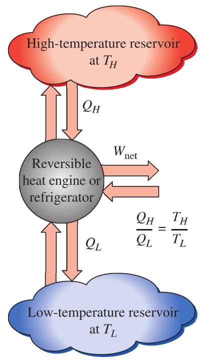
ME 6101: Classical Thermodynamics
`http://zahurul.buet.ac.bd/ME6101/`

Reversibility/Irreversibility

A process commencing from an initial equilibrium state is called **reversible** (or **totally reversible**) if at any time during the process both the system and their environment with which it interacts can be returned to their initial states.



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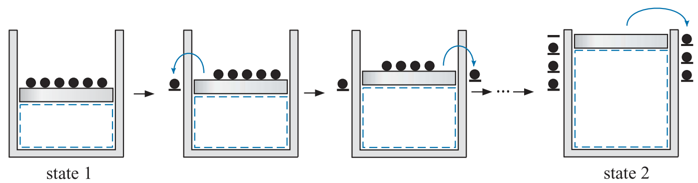
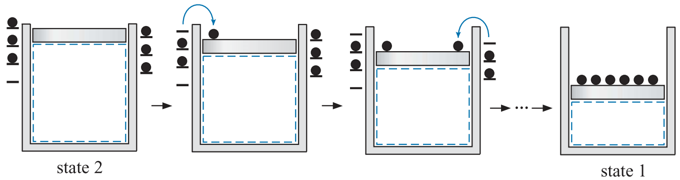
It is the nature of the reversible process that all heat and work interactions which occur during the original (forward) process are equal in magnitude but opposite in direction during the reversed process.



(a)



(b)

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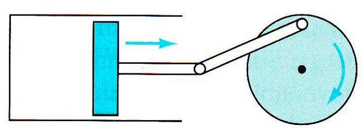
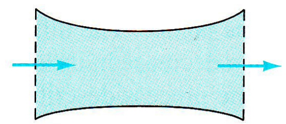
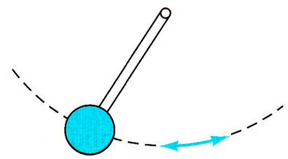
Piston-cylinder apparatus undergoing (a) a slow and incremental expansion and (b) a slow and incremental compression.



(*a*) Piston-flywheel



(*b*) Subsonic nozzle diffuser



(*c*) Ideal pendulum

(*d*) Capacitive-inductive circuit

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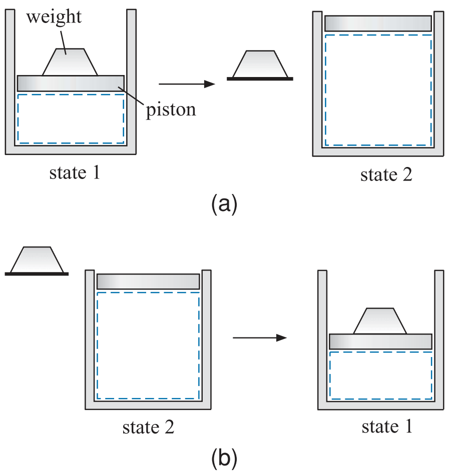
Some reversible processes



T328

Piston-cylinder apparatus undergoing (a) an unconstrained expansion and (b) an unconstrained compression.

Irreversibilities arise from **two** sources:

1. Presence of inherent dissipative effects.
2. Presence of non-quasi-static processes.

Phenomena that identify an irreversible process.

Phenomena	**Unconstrained potential difference**
Friction	Force
Unrestrained expansion	Pressure
Heat transfer across a finite temperature difference	Temperature
Current flow across finite voltage	Voltage
Mixing	Chemical potential

T330

Examples of Irreversible Processes

- Electrical resistance
- Inelastic deformation, internal damping of a vibrating system
- Viscose flow of fluids, throttling, unrestrained expansion of a fluid
- Heat transfer across a finite temperature difference
- Mixing of dissimilar gases and liquids
- Mixing of identical fluids at different temperatures & pressures
- Osmosis
- Hysteresis effects

CM: Reversible & Irreversible Work

- **First Law:** $\delta q - \delta w = du$ **[*reversible* & *actual* cases]**

⇒ $\delta q_{rev} - \delta w_{rev} = du \quad : \quad \delta q_{act} - \delta w_{act} = du$

⇒ $\delta q_{rev} - \delta q_{act} = \delta w_{rev} - \delta w_{act}$

- **Second Law:** $\delta q_{rev} = Tds$
- **Entropy generation:** $\frac{\delta\sigma}{m} \equiv ds - \frac{\delta q_{act}}{T} \geqslant 0$

⇒ $\frac{\delta\sigma}{m} = ds - \frac{\delta q_{act}}{T} = \frac{\delta q_{rev}}{T} - \frac{\delta q_{act}}{T} = \frac{1}{T}(\delta w_{rev} - \delta w_{act}) \geqslant 0$

$$\boxed{\delta w_{rev} - \delta w_{act} = T\frac{\delta\sigma}{m} \quad :\Rightarrow \delta w_{rev} \geqslant \delta w_{act}}$$

$$\boxed{\delta w_{act,in} \geqslant \delta w_{rev,in} \quad : \quad \delta w_{act,out} \leqslant \delta w_{rev,out}}$$

CV: Reversible & Irreversible Work (SSSF)

- **First Law for CV (for actual process):** $0 = \delta q_{act} - \delta w_{act,sf} - \{dh + d(ke) + d(pe)\}$
- **Entropy generation:** $\frac{\delta\sigma}{m} \equiv ds - \frac{\delta q_{act}}{T} \geqslant 0$
- **Maxwell's 2nd relationship:** $dh = Tds + vdP$

$\Rightarrow$ $\delta q_{act} = Tds - T\frac{\delta\sigma}{m} = dh - vdP - T\frac{\delta\sigma}{m}$

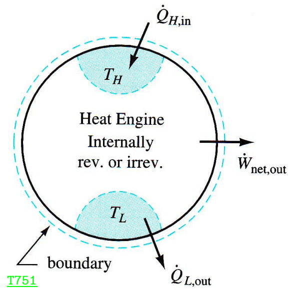
$$\begin{aligned} \Rightarrow \delta w_{act,sf} &= +\delta q_{act} - \{dh + d(ke) + d(pe)\} \\ &= -\{vdP + d(ke) + d(pe) + T\frac{\delta\sigma}{m}\} \end{aligned}$$

$\Rightarrow$ $\boxed{\delta w_{act,sf} = -\{vdP + d(ke) + d(pe) + T\frac{\delta\sigma}{m}\}}$

$\Rightarrow$ $\boxed{\delta w_{rev,sf} = -\{vdP + d(ke) + d(pe)\} \simeq -vdP \quad \because \ \sigma = 0}$

- $\boxed{\delta w_{rev,sf} = \delta w_{act,sf} + T\frac{\delta\sigma}{m}} \Rightarrow \boxed{\delta w_{act,sf} \leqslant \delta w_{rev,sf}}$

Performance of Heat Engine Cycle: Steady State



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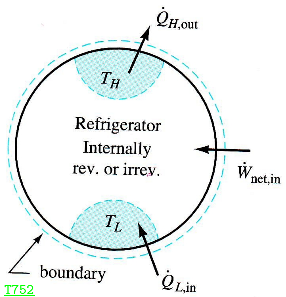
- $\cancel{\frac{dE}{dT}}^{0} = \dot{Q}_{H,in} - \dot{Q}_{L,out} - \dot{W}_{net,out}$
- $\cancel{\frac{dS}{dT}}^{0} = \frac{\dot{Q}_{H,in}}{T_H} - \frac{\dot{Q}_{L,out}}{T_L} + \dot{\sigma}$

$$\begin{aligned} \dot{W}_{net,out} &= \dot{Q}_{H,in} - \dot{Q}_{L,out} \\ &= \dot{Q}_{H,in} - \left[\dot{Q}_{H,in}\frac{T_L}{T_H} + T_L\dot{\sigma}\right] \\ &= \dot{Q}_{H,in}\left[1 - \frac{T_L}{T_H}\right] - T_L\dot{\sigma} \end{aligned}$$

$\Rightarrow$ $\eta_{th} \equiv \frac{\dot{W}_{net,out}}{\dot{Q}_{H,in}} = \left[1 - \frac{T_L}{T_H}\right] - \frac{T_L\dot{\sigma}}{\dot{Q}_{H,in}}$

$\Rightarrow$ $\eta_{th,rev} > \eta_{th,irr} \quad : \dot{\sigma} > 0$

Performance of Refrigeration: Steady State



T752

- $\cancel{\frac{dE}{dT}}^{0} = \dot{Q}_{L,in} - \dot{Q}_{H,out} + \dot{W}_{net,in}$
- $\cancel{\frac{dS}{dT}}^{0} = \frac{\dot{Q}_{L,in}}{T_L} - \frac{\dot{Q}_{H,out}}{T_H} + \dot{\sigma}$

$$\begin{aligned} \dot{W}_{net,in} &= \dot{Q}_{H,out} - \dot{Q}_{L,in} \\ &= \dot{Q}_{L,in}\left[\frac{T_H}{T_L} - 1\right] + T_H\dot{\sigma} \end{aligned}$$

$\Rightarrow$ $COP_R \equiv \frac{\dot{Q}_{L,in}}{\dot{W}_{net,in}} = \left[\frac{T_L}{T_H - T_L}\right] - \frac{T_L T_H}{T_H - T_L}\frac{\dot{\sigma}}{\dot{W}_{net,in}}$

$\Rightarrow$ $COP_{R,rev} > COP_{R,irr} \quad : \dot{\sigma} > 0$

Isolated Process for Incompressible Substances

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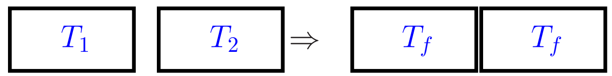


Example: Thermal equilibrium of identical solids.

- Isentropic processes for incompressible substances:

$\Rightarrow$ $\Delta u = \Delta u_1 + \Delta u_2 = 0, \ \rightarrow c_v(T_f - T_1) + c_v(T_2 - T_f) = 0,$

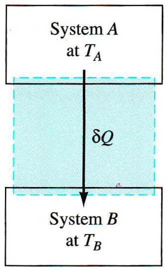
- $\rightarrow T_f = \frac{1}{2}(T_1 + T_2)$

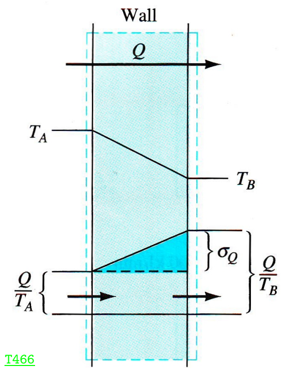
$\Rightarrow$ $ds = \frac{du}{T} = c_v\frac{dT}{T} \rightarrow s_2 - s_1 = c_v \ln\left(\frac{T_2}{T_1}\right)$

$\Rightarrow$ $\Delta s = \Delta s_1 + \Delta s_2 = c_v \ln(T_f/T_1) + c_v \ln(T_f/T_2) = c_v \ln\left[\frac{T_f^2}{T_1 T_2}\right] \geqslant 0$

$\Rightarrow$ If $T_2/T_1 \neq 1, \ \Delta s = +ve$: entropy generation

Entropy Generation in Heat Transfer

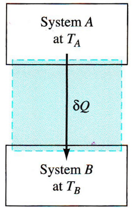




$$\Delta S_{cm} = \sum_{i=1}^{N} \frac{Q_j}{T_j} + \sigma$$

- Steady state, properties are constant $\mapsto \Delta S_{cm} = 0$.
- $\sigma_Q \equiv$ entropy production due to heat transfer.

$\Rightarrow$ $\sigma_Q = -\sum_{j=1}^{N} \frac{\dot{Q}_i}{T_j} = -\left(\frac{Q}{T_A} + \frac{-Q}{T_B}\right) = \frac{Q}{T_A T_B}(T_A - T_B)$.



1. If $T_A > T_B \Rightarrow \sigma_Q > 0$;
2. If $T_A < T_B \rightarrow Q = -ve \Rightarrow \sigma_Q > 0$
3. If T_B is held fixed: $T_A \uparrow \Rightarrow \sigma_Q \uparrow$
4. If $T_A \rightarrow T_B \Rightarrow \sigma_Q \Rightarrow 0$.
5. If $T_A = T_B \Rightarrow \sigma_Q = 0$, reversible heat transfer.

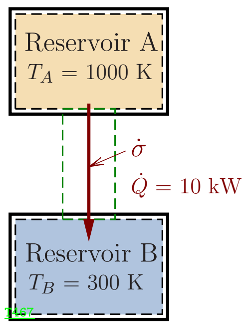
- ΔT must exist for reasonable heat transfer rate to occur, $T_A = T_B + dT$ for reversible heat transfer from A to B.
- Work that can be produced from Q by a reversible heat engine operating between T and T_0: $W_p = Q\,\eta_{carnot} = Q\left(1 - \frac{T_0}{T}\right)$

$\Rightarrow$ **Loss in work potential, $W_{loss,Q}$:**

$$\boxed{W_{loss,Q} = W_{p,A} - W_{p,B} = QT_0\left(\frac{1}{T_B} - \frac{1}{T_A}\right) = T_0\sigma_Q}$$

- The loss in work potential due to irreversible heat transfer is directly proportional to the entropy production in the heat transfer region.

Wark (1999) Ex. 6.9, 6.10: ▷ Estimate (a) entropy production (b) entropy change (c) loss in work potential.



(a) $\dot{\sigma}_Q = \frac{\dot{Q}}{T_A T_B}(T_A - T_B) = 2.333$ kJ/(K.min)

(b) $\frac{dS}{dt} = \frac{dS_A}{dt} + \cancelto{0}{\frac{dS_Q}{dt}} + \frac{dS_B}{dt}$

$\Rightarrow$ $\frac{dS_A}{dt} = -\frac{Q_{A,out}}{T_A} = -1000/1000 = -1.0$ kJ/(K.min)

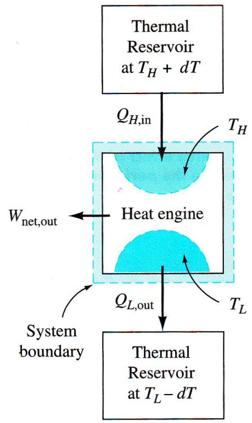
$\Rightarrow$ $\frac{dS_B}{dt} = \frac{Q_{B,in}}{T_B} = +1000/300 = 3.333$ kJ/(K.min)

$\Rightarrow$ $\frac{dS}{dt} = -1.0 + 3.333 = 2.333$ kJ/(K.min)

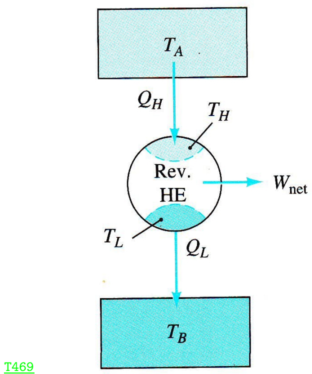
$\Rightarrow$ $\frac{dS}{dt} = \dot{\sigma}_Q = 2.333$ kJ/(K.min)◀

(c) $\dot{W}_{loss,Q} = T_0\dot{\sigma}_Q = 300(2.333) = 700$ kJ/min

Effect of Irreversible HT on HE Cycle Performance

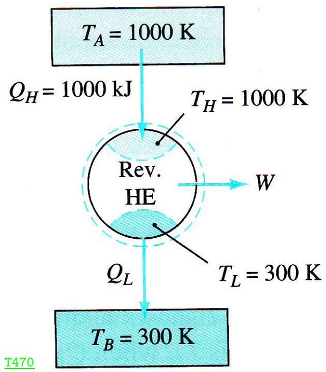


Reversible HE with reversible HT



Reversible HE with irreversible HT

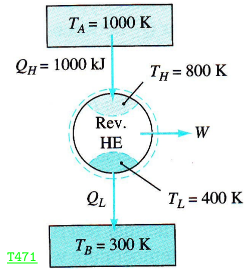
Wark (1999) Ex. 6.14(a): ▷ Reversible HE, operating between T_H & T_L



$\Rightarrow$ $\eta_{th} = 1 - \frac{T_L}{T_H} = 1 - 300/700 = 0.70$

- $W_{net} = Q_{H,in}\eta_{th} = 1000(0.7) = 700$ kJ
- $Q_{L,out} = Q_{H,in} - W_{net} = 300$ kJ
- $\sigma_{Q,H} = \sigma_{L,Q} = 0$
- $\sigma_{HE} = 0$
- Reversible heat transfer $\longrightarrow$ no loss in work potential associated with heat transfer.

Wark (1999) Ex. 6.14(b): ▷



- Reversible HE, operating between T_H & T_L

$\Rightarrow$ $\eta_{th} = 1 - \frac{T_L}{T_H} = 1 - 400/800 = 0.50$

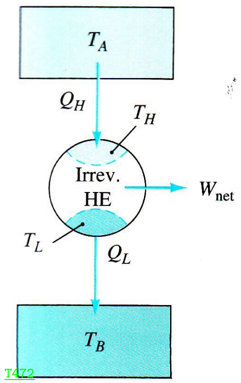
- $W_{net} = Q_{H,in}\eta_{th} = 1000(0.5) = 500$ kJ
- $Q_{L,out} = Q_{H,in} - W_{net} = 1000 - 500 = 500$ kJ
- $\sigma_{HE} = 0$

$\Rightarrow$ $W_{loss,H} = T_0 Q_{H,in}\left(\frac{1}{T_H} - \frac{1}{T_A}\right) = 300(1000)(1/800 - 1/1000) = 75$ kJ

$\Rightarrow$ $W_{loss,L} = T_0 Q_{L,out}\left(\frac{1}{T_B} - \frac{1}{T_L}\right) = 300(500)(1/300 - 1/400) = 125$ kJ

- $W_{loss,net} = 200$ kJ, same as lost work-output between (a) & (b).
- Region of smallest temperature difference (T_L & T_B) produces the largest loss in work potential, although low temperature heat transfer will half the high temperature heat transfer.

Irrev. HE with Irrev. HT



- For engine, $\Delta S_{HE} = \sum_{i=1}^{N} \frac{\dot{Q}_i}{T_i} + \sigma_{HE}$.
- For cyclic engine, $\Delta S_{HE} = 0$.

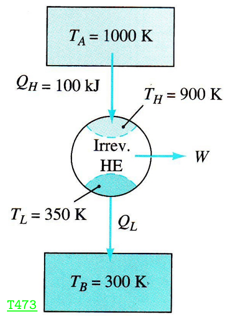
$\Rightarrow$ $\boxed{\sigma_{HE} = -\sum_{i=1}^{N} \frac{\dot{Q}_i}{T_i} = -\left[\frac{Q_{H,in}}{T_H} - \frac{Q_{L,out}}{T_L}\right]}$

- $\sigma_{Q,H} = -\left[\frac{Q_{H,in}}{T_A} - \frac{Q_{H,out}}{T_H}\right]$
- $\sigma_{Q,L} = -\left[\frac{Q_{L,in}}{T_L} - \frac{Q_{L,out}}{T_B}\right]$

$\Rightarrow$ $\boxed{\sigma_{tot} = \sigma_{Q,H} + \sigma_{HE} + \sigma_{Q,L}}$

$\Rightarrow$ $\sigma_{tot} = -\left(\frac{Q_H}{T_H} - \frac{Q_L}{T_L}\right)$, the finding is obvious if a system boundary is drawn around the entire heat engine.

Wark (1999) Ex. 6.15): ▷ given, $\eta_{irrev,HE} = 0.4$



- $W_{net,out} = 0.4(100) = 40$ kJ
- $Q_{L,out} = Q_{H,in} - W_{net,out} = 100 - 40 = 60$ kJ
- $\sigma_{HE} = -\left[\frac{Q_{H,in}}{T_H} - \frac{Q_{L,out}}{T_L}\right] = 0.0603$ kJ
- $\sigma_{Q,H} = -\left[\frac{Q_{H,in}}{T_A} - \frac{Q_{H,out}}{T_H}\right] = 0.0111$ kJ
- $\sigma_{Q,L} = -\left[\frac{Q_{L,in}}{T_L} - \frac{Q_{L,out}}{T_B}\right] = 0.0286$ kJ

$\Rightarrow$ $\sigma_{tot} = \sigma_{Q,H} + \sigma_{HE} + \sigma_{Q,L} = 0.1$ kJ◀