First Law of Thermodynamics

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ME 6101: Classical Thermodynamics

http://zahurul.buet.ac.bd/ME6101/

Conservation of Energy for a CM System

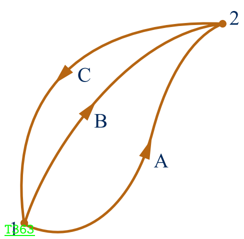
First Law of Thermodynamics (FLT)

When a system undergoes a cyclic change, the net heat to/from the system is equal to the net work from/to the system.

$$J\oint \delta Q = \oint \delta W$$

$$\text{Mechanical equivalent of heat, } J = \begin{cases} 4.1868 \text{ kJ/kcal} \\ 1.0 \text{ in SI unit} \end{cases}$$

First Law of Thermodynamics for a Change in State



$\Rightarrow \oint \delta W = J \oint \delta Q$

$\Rightarrow \oint \delta Q = \oint \delta W \qquad [J = 1.0 \text{ in SI unit}]$

$\Rightarrow \int_1^2 \delta Q_A + \int_2^1 \delta Q_C = \int_1^2 \delta W_A + \int_2^1 \delta W_C \quad ①$

$\Rightarrow \int_1^2 \delta Q_B + \int_2^1 \delta Q_C = \int_1^2 \delta W_B + \int_2^1 \delta W_C \quad ②$

- $① - ② : \int_1^2 \delta Q_A - \int_1^2 \delta Q_B = \int_1^2 \delta W_A - \int_1^2 \delta W_B$

$\Rightarrow \int_1^2 \delta Q_A - \int_1^2 \delta W_A = \int_1^2 \delta Q_B - \int_1^2 \delta W_B$

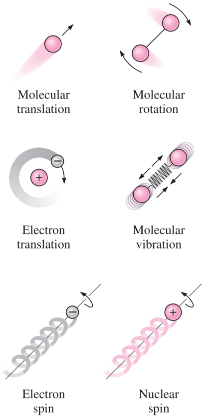
$$\int_1^2 (\delta Q - \delta W)_A = \int_1^2 (\delta Q - \delta W)_B = \cdots$$

$\int_1^2 (\delta Q - \delta W)$ is independent of path and dependent only on the initial and final states; hence, it has the characteristics of a property and this property is denoted by energy, E.

$$\delta Q - \delta W = dE \quad \Rightarrow \quad Q_{12} - W_{12} = \Delta E$$

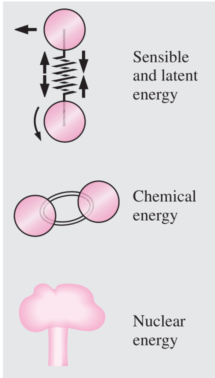
- **Energy (E):** represents all forms energy of the system in the given state. It might be present in a variety of forms, such as:
  - **Kinetic Energy (KE):** energy of a system associated with motion.
  - **Potential Energy (PE):** energy associated with a mass that is located at a specified position in a force field.
  - **Internal Energy (U):** some forms of energy, e.g., chemical, nuclear, magnetic, electrical, and thermal depending in some way on the molecular structure of the substance that is being considered, and these energies are grouped as the internal energy of a system, U.
- KE & PE are external forms of energy as these are independent of the molecular structure of matter. These are associated with the selected coordinate frame and can be specified by the macroscopic parameters of mass, velocity & elevation.
- Internal energy, like kinetic and potential energy, has no natural zero value. So, internal energy of a substance is arbitrarily defined to be zero at some state, known as **Reference State**.

Internal Energy (U): A Thermodynamic Property



T133

Various forms of microscopic energies making up sensible energy.



T134

Internal energy is the sum of all forms of the microscopic energies.

- $$\boxed{E = U + KE + PE + \cdots}$$

$\Rightarrow \delta Q - \delta W = dE = dU + d(KE) + d(PE) + \cdots$

$\Rightarrow \frac{\delta Q}{dt} - \frac{\delta W}{dt} = \frac{dE_{CM}}{dt} = \frac{dU}{dt} + \frac{d(KE)}{dt} + \frac{d(PE)}{dt} + \cdots$

- $$\boxed{\frac{dE_{CM}}{dt} = \frac{dQ}{dt} - \frac{dW}{dt} = \dot{Q} - \dot{W}}$$

$\Rightarrow dU \Rightarrow \int_1^2 dU = U_2 - U_1 = m(u_2 - u_1)$

$\Rightarrow d(KE) = m\mathbb{V}d\mathbb{V} \Rightarrow \int_1^2 d(KE) = \frac{1}{2}m(\mathbb{V}_2^2 - \mathbb{V}_1^2)$

$\Rightarrow d(PE) = mgdZ \Rightarrow \int_1^2 d(PE) = mg(Z_2 - Z_1) = mgh$

$$\boxed{Q_{12} - W_{12} = \left[(U_2 - U_1) + \frac{1}{2}m(\mathbb{V}_2^2 - \mathbb{V}_1^2) + mg(Z_2 - Z_1) + \cdots\right] \simeq (U_2 - U_1)}$$

$$\boxed{q_{12} - w_{12} = \left[(u_2 - u_1) + \frac{1}{2}(\mathbb{V}_2^2 - \mathbb{V}_1^2) + g(Z_2 - Z_1) + \cdots\right] \simeq (u_2 - u_1)}$$

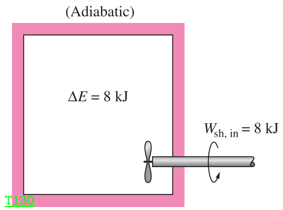
Stationary Systems

$z_1 = z_2 \rightarrow \Delta PE = 0$

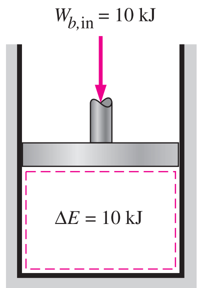
$V_1 = V_2 \rightarrow \Delta KE = 0$

$\Delta E = \Delta U$

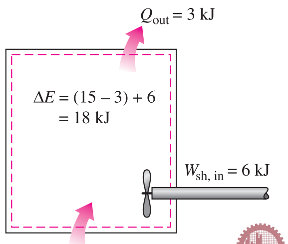
T129



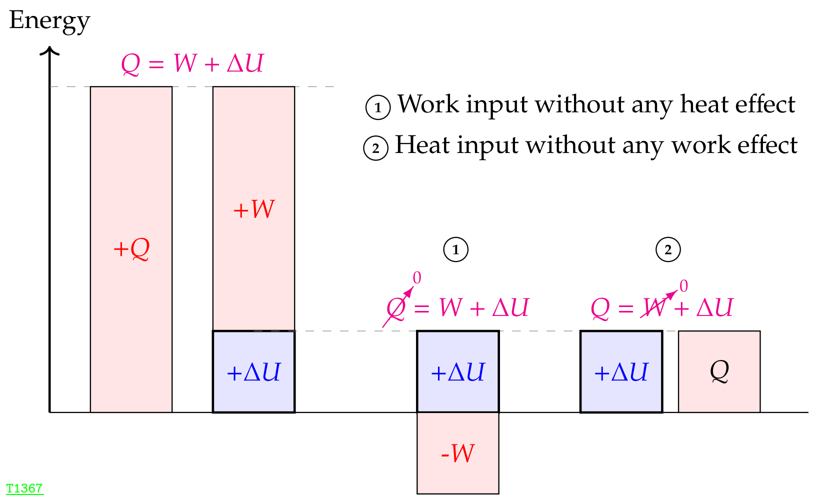
T130



T131 (Adiabatic)



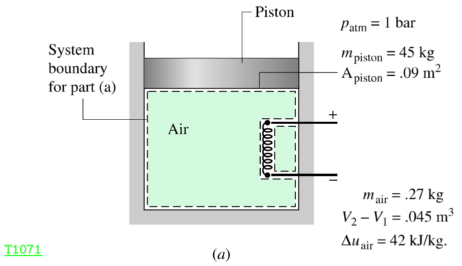
T132



T1367

First Law of Thermodynamics for closed system

Moran Ex. 2.3 ▷ The piston-cylinder material is a ceramic composite and thus a good insulator. Determine the heat transfer from the resistor to the air, in kJ, for a system consisting of (a) the air alone, (b) the air and the piston. [15.8 kJ]



(a): $PA_{piston} = P_{atm}A_{piston} + mg \Rightarrow P = \checkmark$

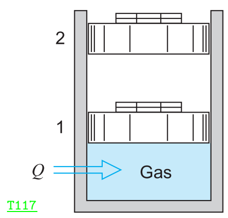
$W_{12} = P(V_2 - V_1) = \checkmark,\ Q_{12} = m(u_2 - u_1) + W_{12} = \checkmark$

(b): $W_{12} = P_{atm}(V_2 - V_1) = \checkmark,\ \Delta PE = m_{piston}gh = m_{piston}g\left(\frac{V_2 - V_1}{A_{piston}}\right) = \checkmark$

$Q_{12} = m(u_2 - u_1) + \Delta PE + W_{12} = \checkmark$

Enthalpy (H): A Thermodynamic Property

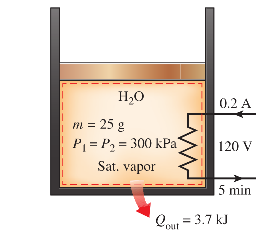
$$H \equiv U + PV \quad \Rrightarrow \quad h \equiv u + Pv$$

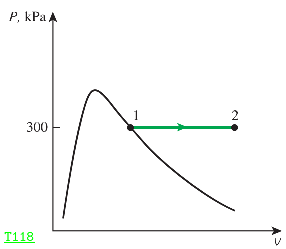


- $\Rightarrow Q_{12} - W_{12} = \Delta E$
- $\Rightarrow Q_{12} - W_{12} = \Delta U \quad$ if $KE \to 0,\ PE \to 0$
- • $W_{12} = \int_1^2 PdV = P(V_2 - V_1)$
- $\Rightarrow Q_{12} = U_2 - U_1 + P(V_2 - V_1)$
- $\Rightarrow Q_{12} = (U_2 + P_2V_2) - (U_1 + P_1V_1)$
- $\Rightarrow Q_{12} = H_2 - H_1$

Heat transfer in a constant-pressure quasi-equilibrium process is equal to the change in enthalpy, which includes both the change in internal energy and the work for this particular process.

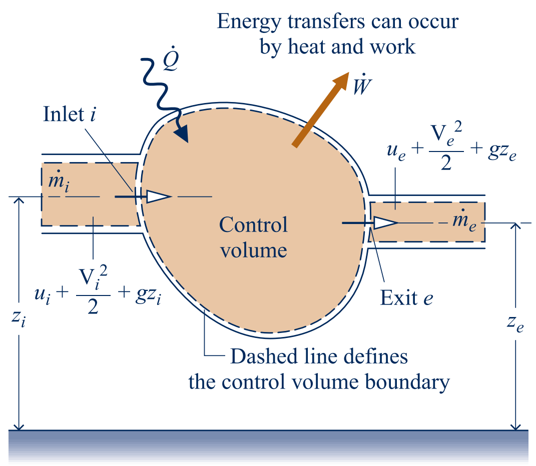
Cengel Ex. 4-5: ▷ Electric Heating of Gas at Constant Pressure: $T_2 = ?$





- $\Rightarrow m = 0.025$kg
- $\Rightarrow Q_{net} = -3.7$ kJ
- $\Rightarrow W_e = -0.2(120)(300)/1000$ kJ
- $\Rightarrow h_1 = h(300\text{ kPa},\ x = 1)$
- $\Rightarrow h_2 = h(300\text{ kPa},\ T_2)$
- $\Rightarrow Q_{net} - W_{net} = \Delta E \simeq \Delta U$
- $\Rightarrow Q_{net} - (W_e + W_b) = \Delta E \simeq \Delta U$
- $\Rightarrow Q_{net} - W_e - P(V_2 - V_1) = U_2 - U_1$
- $\Rightarrow Q_{net} - W_e = m(h_2 - h_1) \to h_2 = \checkmark$
- $\Rightarrow h_2 = h(300\ kPa, T_2) \Rightarrow T_2 = 200^oC \lhd$

Conservation of Energy for CV System



$$\begin{bmatrix}\text{time } \textit{rate of change} \\ \text{of the energy} \\ \text{contained within} \\ \text{the control volume} \\ \textit{at time } t\end{bmatrix} = \begin{bmatrix}\text{net } \textit{rate} \text{ at which} \\ \text{energy is being} \\ \text{transferred in} \\ \text{by heat transfer} \\ \textit{at time } t\end{bmatrix} - \begin{bmatrix}\textit{net} \text{ rate at which} \\ \text{energy is being} \\ \text{transferred out} \\ \text{by work } \textit{at} \\ \textit{time } t\end{bmatrix} + \begin{bmatrix}\textit{net} \text{ rate of energy} \\ \text{transfer } \textit{into} \text{ the} \\ \text{control volume} \\ \text{accompanying} \\ \text{mass flow}\end{bmatrix}$$

T100

$$\begin{aligned}\frac{dE_{cv}}{dt} &= \dot{Q} - \dot{W} + \dot{m}_i e_i - \dot{m}_e e_e \\ &= \dot{Q} - \dot{W} + \dot{m}_i\left(u_i + \frac{\mathbb{V}_i^2}{2} + gz_i\right) - \dot{m}_e\left(u_e + \frac{\mathbb{V}_e^2}{2} + gz_e\right) \\ &= \dot{Q} - \dot{W}_{cv} + \dot{m}_i\left(h_i + \frac{\mathbb{V}_i^2}{2} + gz_i\right) - \dot{m}_e\left(h_e + \frac{\mathbb{V}_e^2}{2} + gz_e\right)\end{aligned}$$

- $\dot{W} = \dot{W}_s + \dot{W}_b + \dot{W}_f = \dot{W}_{cv} + \dot{W}_f$
- $\dot{W}_f = -P(\dot{V}_i - \dot{V}_e) = -P(\dot{m}_i v_i - \dot{m}_e v_e)$
- $h \equiv u + Pv$

First Law of Thermodynamics (FLT) for CV System

$$\frac{dE_{CV}}{dt} = \dot{Q} - \dot{W}_{cv} + \sum_i \dot{m}_i\left(h_i + \frac{\mathbb{V}_i^2}{2} + gz_i\right) - \sum_e \dot{m}_e\left(h_e + \frac{\mathbb{V}_e^2}{2} + gz_e\right)$$

- **Closed System**: $\mapsto \dot{m}_i = \dot{m}_e = 0$.

$$\frac{dE_{CM}}{dt} = \dot{Q} - \dot{W}_{net}$$

- **Closed & Adiabatic (Isolated) System**: $\mapsto \dot{m}_i = \dot{m}_e = 0,\ \dot{Q} = 0$.

$$\frac{dE_{CM}}{dt} = -\dot{W}_{net} \Rightarrow \Delta E_{CM} = -W_{ad}$$

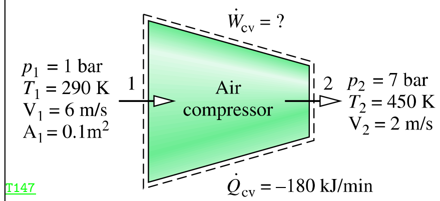
- **Steady-State-Steady Flow (SSSF) System**:

$$\frac{dm_{CV}}{dt} = 0 \Rightarrow \sum_i \dot{m}_i = \sum_e \dot{m}_e \quad : \quad \frac{dE_{cv}}{dt} = 0$$

- **One-inlet, One-exit & Steady-state**: $\mapsto \dot{m}_i = \dot{m}_e = \dot{m}$.

$$0 = \dot{Q} - \dot{W}_{CV} + \dot{m}\left[(h_1 - h_2) + \left(\frac{\mathbb{V}_1^2 - \mathbb{V}_2^2}{2}\right) + g(z_1 - z_2)\right]$$

[Moran Ex. 4.5]: ▷ Air Compressor Power: Determine power required, $\dot{W}_{cv}$.



T147

- ⇒ Steady state ⇒ $dE_{cv}/dt = 0$
- ⇒ $Z_2 = Z_1$
- ⇒ $\dot{Q}_{cv} = -180 kJ/min = -3.0$ kW
  - $\dot{m} = \rho A \mathbb{V}$
  - $\rho = P/RT$

$$\frac{dE_{CV}}{dt} = \dot{Q} - \dot{W}_{cv} + \sum_i \dot{m}_i\left(h_i + \frac{\mathbb{V}_i^2}{2} + gz_i\right) - \sum_e \dot{m}_e\left(h_e + \frac{\mathbb{V}_e^2}{2} + gz_e\right)$$

$$0 = \dot{Q} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \left(\frac{\mathbb{V}_1^2 - \mathbb{V}_2^2}{2}\right)\right]$$

⇒ $h_1 - h_2 = C_p(T_1 - T_2) = -160.8$ kJ/kg

⇒ $\rho_1 = P_1/RT_1 = 1.20$ m³/kg, $\dot{m} = \rho_1 A_1 \mathbb{V}_1 = 0.72$ kg/s

⇒ $\dot{W}_{cv} = -119.4$ kW (work input required) ◁

Bernoulli's Equation

- $h = u + Pv \rightarrow dh = du + Pdv + vdP$, so for isothermal process ($du = 0$) and incompressible fluid ($dv = 0$):

⇒ $dh = vdP \rightsquigarrow$ $h_2 - h_1 = v(P_2 - P_1) = \frac{P_2 - P_1}{\rho}$

- For a steady state flow device if $\Delta PE \neq 0$, $\Delta KE \neq 0$, $W_{cv} = 0$ and $Q_{cv} = 0$:

⇒ $0 = 0 - 0 + \dot{m}\left[(h_1 - h_2) + \left(\frac{\mathbb{V}_1^2 - \mathbb{V}_2^2}{2}\right) + g(z_1 - z_2)\right]$

$$\frac{P_1}{\rho g} + \frac{\mathbb{V}_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{\mathbb{V}_2^2}{2g} + z_2$$

$\frac{P}{\rho g}$: pressure head

$\frac{\mathbb{V}^2}{2g}$: velocity head

z : elevation head

First Law of Thermodynamics: Transient Analysis

$$dE_{CV} = \delta Q - \delta W_{cv} + \sum_{in} \left(h + \frac{\mathbb{V}^2}{2} + gz\right)_i dm_i - \sum_{out} \left(h + \frac{\mathbb{V}^2}{2} + gz\right)_e dm_e$$

$$\int_0^t \left(\frac{dE_{CV}}{dt}\right) dt = \int_0^t (\dot{Q} - \dot{W}_{cv}) dt + \sum_{in} \int_0^t \dot{m}_i e_i \, dt - \sum_{out} \int_0^t \dot{m}_e e_e \, dt$$

$$e \equiv h + \frac{\mathbb{V}^2}{2} + gz$$

⇒ If CV is fixed in space, $E_{cv} = U_{cv}$

⇒ $\int \left(\frac{dE_{cv}}{dt}\right) dt = \int dU_{cv} = \int d(mu) = m_2 u_2 - m_1 u_1 = \Delta U_{cv}$

⇒ $\int dU_{cv} = \int d(mu) = \int (mdu + udm)$: alternative form.

⇒ $\int dU_{cv} = Q - W_{cv} + \int \left(h + \frac{\mathbb{V}^2}{2} + gz\right)_i dm_i - \int \left(h + \frac{\mathbb{V}^2}{2} + gz\right)_e dm_e$

In the analysis of transient flow system, **two** models are widely used:

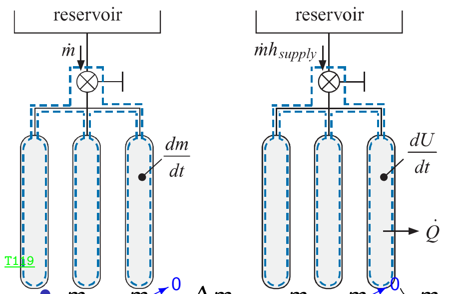
1. **Uniform State:** state within CV at any instant is uniform throughout the CV. However, the state within CV may change with time. It implies rapid or instantaneous approach to equilibrium at all times for the mass within CV.
2. **Uniform Flow:** state of mass crossing a CS is invariant with time. However, mass flow rate across that particular CS may vary with time. This condition is frequently met when the flow into transient system is supplied from a very large reservoir.

$$\Delta m_{CV} = m_2 - m_1 = m_i - m_e$$

$$\Delta E_{CV} = m_2 u_2 - m_1 u_1 = Q - W_{CV} + m_i e_i - m_e e_e$$

Charging of Evacuated Vessel

Mass enters only at one section of CS and no efflux of matter.



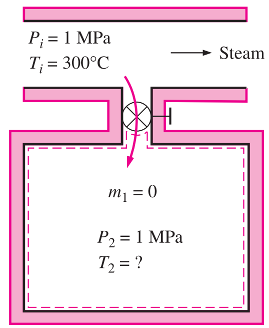
Assumptions:

- Uniform inlet flow
- Uniform state within the tank at any instant
- Rigid tank and $W_{cv} = 0$
- Effects of KE & PE → 0.

- $m_2 - \cancelto{0}{m_1} = \Delta m_{cv} = m_i - \cancelto{0}{m_e} \Rightarrow m_2 = m_i$
- If tank is insulated or filled rapidly, $Q \longrightarrow 0$
- $\Delta U_{cv} = \cancelto{0}{Q} - \cancelto{0}{W_{cv}} + \int e_i dm_i - \int e_e \cancelto{0}{dm_e} \longrightarrow m_2 u_2 = m_i h_i = m_2 h_i$

⇒ $\boxed{u_2 = h_i}$

Cengel Ex. 5-12 ▷ Charging of evacuated Vessel



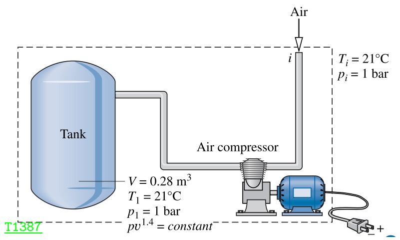
T120

⇒ $h_i = h(Steam,\ P_1 = 1.0\ MPa,\ T_1 = 300^oC) = 3051$ kJ/kg

⇒ $h_i = u_2 = u(Steam,\ P_2 = 1.0\ MPa,\ T_2 = ?) \longmapsto T_2 = 456^oC$ ◁

- **What happens in case of CNG cylinder filling?** $u_2 = h_i \rightarrow T_2 = kT_i$

Moran Ex.4-13 ▷ Charging of a vessel: Estimate work input required if final pressure is 4 bar.

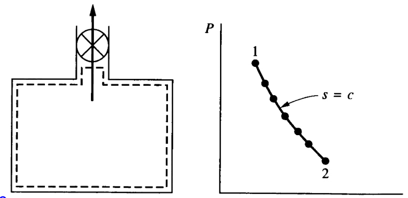


T1387

- $V = 0.23$ m^3
- $R = 287$ J/kg-K
- $P_1 = P_i = 10^5$ Pa
- $P_2 = 4 \times 10^5$ Pa

- $PV = mRT$; $V = m_1 v_1 = m_2 v_2$; $P_1 v_1^{1.4} = P_2 v_2^{1.4}$
- $\Delta U_{cv} = -W_{cv} + h_i \Delta m_{cv}$, for uniform flow, pipeline condition is unchanged.
- $m_1 = 0.332$, $m_2 = 0.893$ kg
- $T_2 = 163.96$ °C
- $W_{cv} = h_i \Delta m_{cv} - \Delta U_{cv} = h_i(m_2 - m_1) - (m_2 u_2 - m_1 u_1) = 44.7$ kJ/kg

Discharging Process



e835

$\Rightarrow dm_{cv} = dm_i^{\nearrow 0} - dm_e = 0 - dm_e \Rrightarrow dm_{cv} = -dm_e$

$\Rightarrow dE_{cv} = \delta Q - \delta W_{cv} + \left(h + \frac{\mathbb{V}^2}{2} + gz\right)_i dm_i^{\nearrow 0} - \left(h + \frac{\mathbb{V}^2}{2} + gz\right)_e dm_e$

$\Rightarrow dU_{cv} = -h_e dm_e = +hdm : \; m = m_{cv} =$ mass in CV

$\Rightarrow hdm = dU_{cv} = d(mu) = mdu + udm$

$\Rightarrow (h - u)dm = mdu \longrightarrow \frac{dm}{m} = \frac{du}{h-u} = \frac{du}{Pv}$

$\Rightarrow V = mv \rightarrow dV^{\nearrow 0} = mdv + vdm \Rightarrow -\frac{dv}{v} = \frac{dm}{m}$

$\Rightarrow \frac{du}{Pv} = \frac{dm}{m} = -\frac{dv}{v} \Rightarrow \underbrace{du + Pdv}_{Tds} = 0 \rightarrow ds = 0 \Rightarrow \boxed{s = const}$

Example: ▷ Air ($P_1 = 10$ MPa, $T_1 = 25°$ C) is allowed to escape from a vessel of 20 m^3 to 200 KPa. Estimate final temperature.

$\Rightarrow s =$ constant $\Rightarrow Pv^k =$ constant $\Rightarrow \frac{dP}{P} + k\frac{dv}{v} = 0$

$\Rightarrow \frac{dm}{m} = -\frac{dv}{v} = \frac{1}{k}\frac{dP}{P} \Rightarrow \frac{m_2}{m_1} = \left(\frac{P_2}{P_1}\right)^{1/k} = \left(\frac{T_2}{T_1}\right)^{1/(k-1)}$

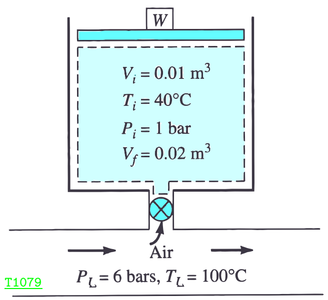
$\Rightarrow m_1 = \frac{P_1 V_1}{RT_1} = 2338$ kg

$\Rightarrow \frac{m_2}{m_1} = \left(\frac{P_2}{P_1}\right)^{1/k} \Rightarrow m_2 = 143$ kg

$\Rightarrow m_2 = \frac{P_2 V_2}{RT_2} \Rightarrow T_2 = -175°C \triangleleft$

Wark Ex. 5-15: ▷ If cylinder pressure remains constant, estimate: (a) mass entered through the valve, (b) final temperature. [9.3 g, 342 K]

- $m = \frac{PV}{RT}$
- $\Delta m_{CV} = m_2 - m_1 = m_i - m_e$
- $\Delta E_{CV} = m_2 u_2 - m_1 u_1 = Q - W_{CV} + m_i e_i - m_e e_e$



T1079

- air: $c_p = 1000$, $c_v = 720$ J/kgK
- $m_1 = PV_1/RT_1 = \surd$, $m_2 = PV_2/RT_2$
- $m_e = 0 \rightarrow m_2 - m_1 = m_i$
- $Q = 0$, $W_{cv} = P(V_2 - V_1) = \surd$
- $e_i = h_i \simeq c_p T_i = \surd$
- $u \simeq c_v T$, $u_1 = \surd$, $u_2 = c_v T_2$
- Energy Equation: $f(T_2)$ only, $\rightarrow T_2 = \surd$
- $m_i = m_2 - m_1 = \surd$