System Stability

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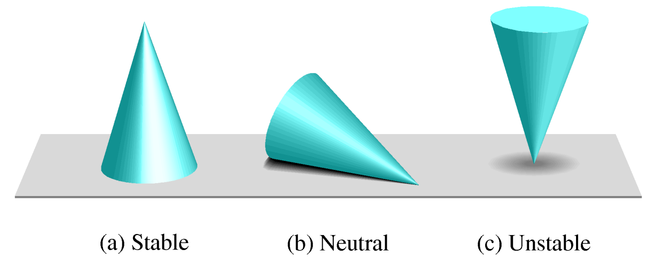
ME 475: Mechatronics

System Stability

A **Stable** system is a dynamic system with a bounded response to a bounded input.



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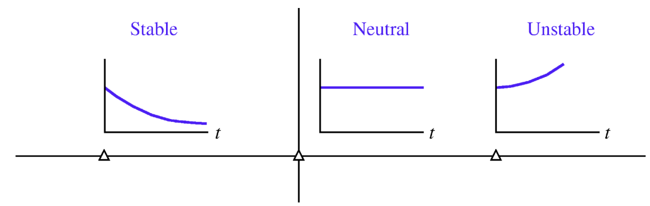

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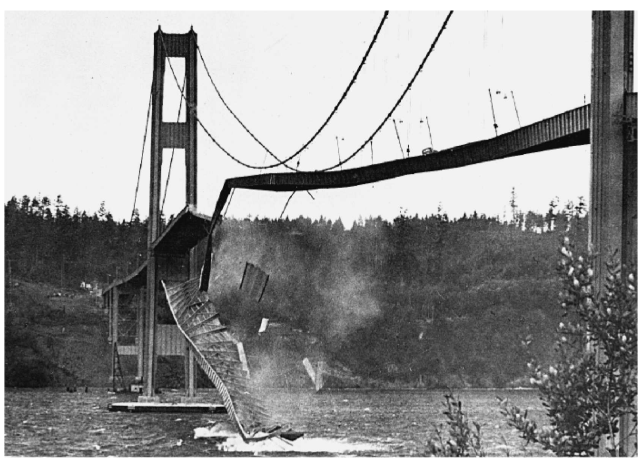
System Instability: Example



e133.eps

Tacoma Narrows bridge at catastrophic failure

Routh-Hurwitz Criterion for Stability

The method involves two steps:

1. Generate a data table called a Routh table.
2. Interpret the Routh table to tell how many system-poles are in the left half-plane, the right half-plane, and on the $j\omega$-axis.

The Routh-Hurwitz Criterion declares that:
the number of roots of the polynomial that are on the right half-plane is equal to the number of sign changes of the first column of the Routh table.

Generation of a Basic Routh Table

$$R(s) \longrightarrow \boxed{\frac{N(s)}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}} \longrightarrow C(s)$$

s^4	a_4	a_2	a_0
s^3	a_3	a_1	0
s^2			
s^1			
s^0			

e134.eps

Initial layout for Routh table

s^4	a_4	a_2	a_0
s^3	a_3	a_1	0
s^2	$\dfrac{-\begin{vmatrix} a_4 & a_2 \\ a_3 & a_1 \end{vmatrix}}{a_3} = b_1$	$\dfrac{-\begin{vmatrix} a_4 & a_0 \\ a_3 & 0 \end{vmatrix}}{a_3} = b_2$	$\dfrac{-\begin{vmatrix} a_4 & 0 \\ a_3 & 0 \end{vmatrix}}{a_3} = 0$
s^1	$\dfrac{-\begin{vmatrix} a_3 & a_1 \\ b_1 & b_2 \end{vmatrix}}{b_1} = c_1$	$\dfrac{-\begin{vmatrix} a_3 & 0 \\ b_1 & 0 \end{vmatrix}}{b_1} = 0$	$\dfrac{-\begin{vmatrix} a_3 & 0 \\ b_1 & 0 \end{vmatrix}}{b_1} = 0$
s^0	$\dfrac{-\begin{vmatrix} b_1 & b_2 \\ c_1 & 0 \end{vmatrix}}{c_1} = d_1$	$\dfrac{-\begin{vmatrix} b_1 & 0 \\ c_1 & 0 \end{vmatrix}}{c_1} = 0$	$\dfrac{-\begin{vmatrix} b_1 & 0 \\ c_1 & 0 \end{vmatrix}}{c_1} = 0$

e135.eps

Completed Routh table

Stability Analysis: R-W Criterion

Example: $R(s) \longrightarrow \boxed{\frac{1000}{s^3 + 10s^2 + 31s + 1030}} \longrightarrow C(s)$

s^3	1	31	0
s^2	$\cancel{10}\ 1$	$\cancel{1030}\ 103$	0
s^1	$\dfrac{-\begin{vmatrix} 1 & 31 \\ 1 & 103 \end{vmatrix}}{1} = -72$	$\dfrac{-\begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix}}{1} = 0$	$\dfrac{-\begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix}}{1} = 0$
s^0	$\dfrac{-\begin{vmatrix} 1 & 103 \\ -72 & 0 \end{vmatrix}}{-72} = 103$	$\dfrac{-\begin{vmatrix} 1 & 0 \\ -72 & 0 \end{vmatrix}}{-72} = 0$	$\dfrac{-\begin{vmatrix} 1 & 0 \\ -72 & 0 \end{vmatrix}}{-72} = 0$

e136.eps

Two sign changes in the first column, system is **unstable** since two poles exists in the right half-plane.

Special Cases of R-W Criterion

1. *Table have a zero only in the first column of a row.* In such case, an epsilon, ϵ, is assigned to replace the zero and the value of ϵ is then allowed to approach zero from either the positive or negative side after which the signs of the entries of the first column can be determined.
2. *Table have an entire row that consists of zeros.* In such case, we return to the row immediately above the row of zeros and form an auxiliary polynomial, using the entries in that row as coefficients. The polynomial is then differentiated with respect to s to obtain the coefficients to replace the rows of zeros.

Example of a R-W Criterion: Case 1

$$R(s) \longrightarrow \boxed{\frac{10}{s^5+2s^4+3s^3+6s^2+5s+3}} \longrightarrow C(s)$$

s^5	1	3	5
s^4	2	6	3
s^3	$\cancel{0}\ \epsilon$	$\frac{7}{2}$	0
s^2	$\frac{6\epsilon - 7}{\epsilon}$	3	0
s^1	$\frac{42\epsilon - 49 - 6\epsilon^2}{12\epsilon - 14}$	0	0
s^0	3	0	0

e137.eps

Label	First Column	$\epsilon = +$	$\epsilon = -$
s^5	1	+	+
s^4	2	+	+
s^3	$\cancel{0}\ \epsilon$	+	−
s^2	$\frac{6\epsilon - 7}{\epsilon}$	−	+
s^1	$\frac{42\epsilon - 49 - 6\epsilon^2}{12\epsilon - 14}$	+	+
s^0	3	+	+

e138.eps

The system is unstable, with two poles in the right half-plane

Example of a R-W Criterion: Case 2

$$R(s) \longrightarrow \boxed{\frac{10}{s^5+7s^4+6s^3+42s^2+8s+56}} \longrightarrow C(s)$$

s^5	1	6	8
s^4	$\cancel{7}\ 1$	$\cancel{42}\ 6$	$\cancel{56}\ 8$
s^3	$\cancel{0}\ \cancel{4}\ 1$	$\cancel{0}\ \cancel{12}\ 3$	$\cancel{0}\ \cancel{0}\ 0$
s^2	3	8	0
s^1	$\frac{1}{3}$	0	0
s^0	8	0	0

e139.eps

No sign change, stable system

Case 2: Procedure Steps

1. Start by forming Routh table.
2. At 2^{nd} row, multiply through 1/7 for convenience.
3. At 3^{rd} row, entire row consists of zeros; then
   - return to the row immediately above the row consisting zeros,
   - form an auxiliary polynomial, using the entries in that row,

$$P(s) = s^4 + 6s^2 + 8 \Longrightarrow \frac{dP(s)}{ds} = 4s^3 + 12s + 0$$

   - the coefficients [4,12,0] are then used to replace the zeros,
   - for convenience, the 3^{rd} row is multiplied by 1/4
4. Remain of the table is formed in standard manner.