Numbering System

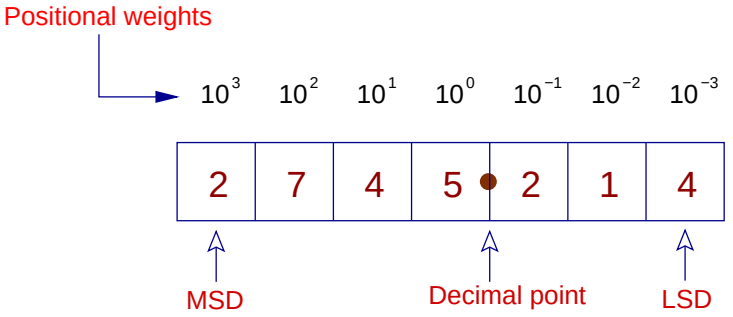
Prof. Dr. M. Zahurul Haq
zahurul@me.buet.ac.bd
http://teacher.buet.ac.bd/zahurul/

Department of Mechanical Engineering
Bangladesh University of Engineering & Technology

ME 361: Instrumentation & Measurement

Decimal Numbering System (Base 10)

Decimal system is composed of 10 digits:
0, 1, 2, 3, 4, 5, 6, 7, 8 & 9.

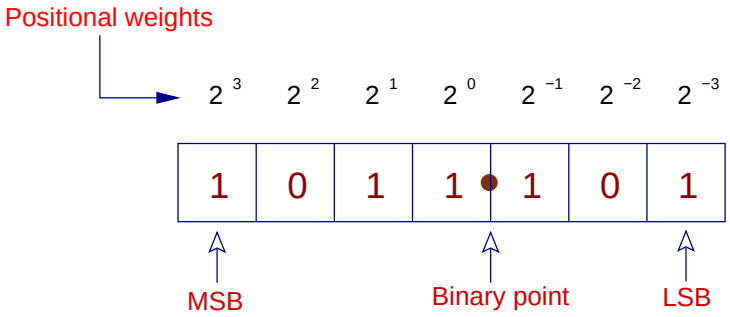


x050.eps

$$2745.214_{10} = 2\cdot 10^3 + 7\cdot 10^2 + 4\cdot 10^1 + 5\cdot 10^0 + 2\cdot 10^{-1} + 1\cdot 10^{-2} + 4\cdot 10^{-3}$$

Binary Numbering System (Base 2)

In binary system there are only two digit values: 0 & 1.



x051.eps

$$\begin{aligned} 1011.101_2 &= 1\cdot 2^3 + 0\cdot 2^2 + 1\cdot 2^1 + 1\cdot 2^0 + 1\cdot 2^{-1} + 0\cdot 2^{-2} + 1\cdot 2^{-3} \\ &= 8 + 0 + 2 + 1 + 0.5 + 0 + 0.125 = 11.625_{10} \end{aligned}$$

Decimal $\Longleftrightarrow$ Binary Conversion

$14.375_{10} = ?$

- Step 1: $14_{10} = ?_2$

14	$\div$ 2	=	7	remainder 0	LSB
7	$\div$ 2	=	3	remainder 1	
3	$\div$ 2	=	1	remainder 1	
1	$\div$ 2	=	0	remainder 1	MSB

$\Longrightarrow 14_{10} = 1110_2$

- Step 2: $0.375_{10} = ?_2$

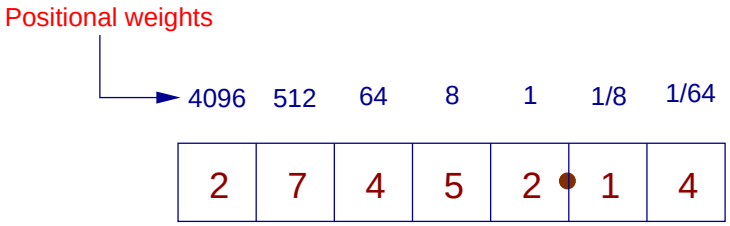
0.375	$\times$ 2	= 0.75	=	0.75	with a carry of 0
0.75	$\times$ 2	= 1.50	=	0.50	with a carry of 1
0.50	$\times$ 2	= 1.00	=	0.00	with a carry of 1

$\Longrightarrow 0.375_{10} = 011_2$

$\Longrightarrow 14.375_{10} = 1110.011_2$

Octal Numbering System (Base 8)

In octal number system uses 8 digit symbols:
0, 1, 2, 3, 4, 5, 6 & 7.



x052.eps

$$\begin{aligned} 27452.14_8 &= 2\cdot 8^4 + 7\cdot 8^3 + 4\cdot 8^2 + 5\cdot 8^1 + 2\cdot 8^0 + 1\cdot 8^{-1} + 4\cdot 8^{-2} \\ &= 8192 + 3584 + 256 + 40 + 2 + 0.125 + 0.0625 \\ &= 12074.1875_{10} \end{aligned}$$

Octal Number Conversions

- Octal to binary is performed by converting each octal digit to its 3-bit binary equivalent.
- Example: binary to octal conversion

$$10111001_2 = \underbrace{010}_{2}\ \underbrace{111}_{7}\ \underbrace{001}_{1} = 271_8$$

- Example: octal to binary conversion

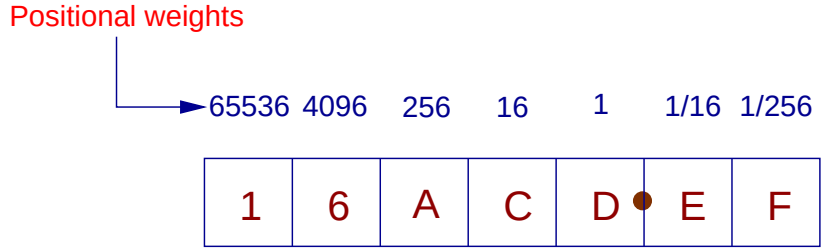
$$624_8 = \overbrace{110}^{6}\ \overbrace{010}^{2}\ \overbrace{100}^{4} = 110010100_2$$

- Example: $486_{10} = ?_8$

$$\left.\begin{array}{rcrcll} 486 & \div & 8 & = 60 & \textit{remainder} & 6 \\ 60 & \div & 8 & = 7 & \textit{remainder} & 4 \\ 7 & \div & 8 & = 0 & \textit{remainder} & 7 \end{array}\right\} = 746_8$$

Hexadecimal Numbering (Base 16)

- 16 allowable digits are:
  0, 1, 2, 3,4, 5, 6, 7, 8, 9, A, B, C, D, E & F.
- to signify hex digits, a subscript 16 or a letter H is used ($A7_{16} = A7H$).
- Four bits (one hex digit) is known as *nibble*.



x053.eps

$16ACD.EF_{16} = 92877.93359_{10}$

Hexadecimal Number Conversions

- Hexadecimal to binary is performed by converting each octal digit to its 4-bit binary equivalent.
- Example: binary to hexadecimal conversion

$$01101101_2 = \underbrace{0110}_{6}\ \underbrace{1101}_{D} = 6D_{16}$$

- Example: octal to binary conversion

$$A9H = \overbrace{1010}^{A}\ \overbrace{1001}^{9} = 10101001_2$$

- Example: $498_{10} = ?_{16}$

$$\left.\begin{array}{rcrcll} 498 & \div & 16 & = 31 & \textit{remainder} & 2 \\ 31 & \div & 16 & = 1 & \textit{remainder} & 15 = F \\ 1 & \div & 16 & = 0 & \textit{remainder} & 1 \end{array}\right\} = 1F2_{16}$$

Binary Coded Decimal (BCD) Number

- Digital systems use some form of binary numbers for their internal operations, but the external world is decimal in nature.
- In BCD system, each digit of a decimal number is replaced by its 4-bit binary equivalent, and vice versa.
- Example: decimal to BCD conversion

$$496_{10} = \overbrace{0100}^{4}\ \overbrace{1001}^{9}\ \overbrace{0110}^{6} = 010010010110_{BCD}$$

- Example: BCD to decimal conversion

$$011101011000_{BCD} = \underbrace{0111}_{7}\ \underbrace{0101}_{5}\ \underbrace{1000}_{8} = 758_{10}$$

Comparison of Numbering Systems

Decimal	Binary	Octal	Hexadecimal	BCD
0	00000000	00	00	00000000
1	00000001	01	01	00000001
2	00000010	02	02	00000010
3	00000011	03	03	00000011
4	00000100	04	04	00000100
5	00000101	05	05	00000101
6	00000110	06	06	00000110
7	00000111	07	07	00000111
8	00001000	10	08	00001000
9	00001001	11	09	00001001

Numbering Systems . . . contd.

Decimal	Binary	Octal	Hexadecimal	BCD
10	00001010	12	0A	00010000
11	00001011	13	0B	00010001
12	00001100	14	0C	00010010
13	00001101	15	0D	00010011
14	00001110	16	0E	00010100
15	00001111	17	0F	00010101
16	00010000	20	10	00010110
17	00010001	21	11	00010111
18	00010010	22	12	00011000
19	00010011	23	13	00011001
20	00010100	24	14	00100000