

## Ideal Gas Mixtures

Dr. Md. Zahurul Haq, *Ph.D., CEA, FBSME, FIEB*

Professor  
Department of Mechanical Engineering  
Bangladesh University of Engineering & Technology (BUET)  
Dhaka-1000, Bangladesh

zahurul@me.buet.ac.bd  
http://zahurul.buet.ac.bd/

ME 407: Advanced Thermodynamics  
http://zahurul.buet.ac.bd/ME407/



## Overview

### 1 Mixtures of Ideal Gas

### 2 Psychrometry



## Ideal Gas Mixtures

$$\bullet m = m_1 + m_2 + m_3 + \dots + m_k = \sum_{i=1}^k m_i$$

$$mf_i \equiv \frac{m_i}{m} \quad \Rightarrow \quad \sum_{i=1}^k mf_i = 1$$

$$\bullet n = n_1 + n_2 + n_3 + \dots + n_k = \sum_{i=1}^k n_i$$

$$Y_i \equiv \frac{n_i}{n} \quad \Rightarrow \quad \sum_{i=1}^k Y_i = 1$$

$m_i$  = mass of component  $i$

$m$  = total mass of mixture

$n_i$  = number of moles of component  $i$

$n$  = total number of moles in mixture

$mf_i$  = mass fraction of component  $i$

$Y_i$  = mole fraction of component  $i$

$M_i$  = molecular mass of component  $i$

$M$  = apparent molecular mass of mixture  $i$



$$m_i = n_i M_i \quad ; \quad m = nM$$

$$\bullet M = \frac{m}{n} = \frac{m_1 + m_2 + \dots + m_k}{n} = \frac{n_1 M_1 + n_2 M_2 + \dots + n_k M_k}{n} = \sum_{i=1}^k \left( \frac{n_i}{n} \right) M_i$$

$$M = \sum_{i=1}^k Y_i M_i$$

Dry air: 78.08%  $N_2$ , 20.95%  $O_2$ , 0.93%  $Ar$ , 0.03%  $CO_2$ :

$$M = 0.7808 \cdot 28 + 0.2095 \cdot 32 + 0.0093 \cdot 39.94 + 0.0003 \cdot 44 = 28.95 \text{ kg/kmol}$$

$$\bullet \text{ Apparent gas constant, } R = \frac{R_u}{M}$$

$$\text{For air: } R = \frac{8.314}{28.95} = 0.287 \text{ kJ/kg K}$$

$$\bullet M = \frac{m}{n} = \frac{m}{\sum_{i=1}^k n_i} = \frac{m}{\sum_{i=1}^k \left( \frac{m_i}{M_i} \right)} = \frac{1}{\sum_{i=1}^k \left( \frac{mf_i}{M_i} \right)}$$

$$M = \frac{1}{\sum_{i=1}^k \left( \frac{mf_i}{M_i} \right)}$$

$$\bullet mf_i = \frac{m_i}{m} = \frac{n_i M_i}{\sum_{i=1}^k n_i M_i} = \frac{n_i M_i / n}{\sum_{i=1}^k Y_i M_i} \quad \Rightarrow \quad mf_i = \frac{Y_i M_i}{M}$$

$$mf_i = \frac{Y_i M_i}{M} = \frac{Y_i M_i}{\sum_{i=1}^k Y_i M_i} \quad ; \quad Y_i = \left( \frac{mf_i}{M_i} \right) M = \frac{mf_i / M_i}{\sum_{i=1}^k mf_i / M_i}$$



Conversion: Mass fraction ( $mf_i$ ) to Mole Fraction ( $Y_i$ )

$$Y_i = M \left[ \frac{mf_i}{M_i} \right] = \frac{mf_i/M_i}{\sum mf_i/M_i}$$

$i$	$mf_i$	$M_i$	$\frac{mf_i}{M_i}$	$Y_i = \frac{mf_i/M_i}{\sum mf_i/M_i}$
$H_2$	0.10	2.0	0.050	0.6250
$O_2$	0.48	32.0	0.015	0.1875
$CO$	0.42	28.0	0.015	0.1875
	1.00	-	0.080	1.0000

$$M = \frac{1}{\sum mf_i/M_i} = \frac{1}{0.080} = 12.5 \text{ kg/kmol.}$$

- If  $m_i$ 's are given:

- $m_i \rightarrow mf_i = \frac{m_i}{\sum m_i} = \frac{m_i}{m}$
- $n = \frac{m}{M}$
- $n_i = Y_i n$



## Conversion: Mole fraction to Mass Fraction

$$mf_i = \frac{Y_i M_i}{M} = \frac{Y_i M_i}{\sum Y_i M_i}$$

$i$	$Y_i$	$M_i$	$Y_i M_i$	$mf_i = \frac{Y_i M_i}{\sum Y_i M_i}$
$H_2$	0.6250	2.0	1.25	0.10
$O_2$	0.1875	32.0	6.00	0.48
$CO$	0.1875	28.0	5.25	0.42
	1.00	-	12.5	1.0000

$$M = \sum Y_i M_i = 12.5 \text{ kg/kmol.}$$

- If  $n_i$ 's are given:

- $n_i \rightarrow Y_i = \frac{n_i}{\sum n_i} = \frac{n_i}{n}$
- $m = nM$
- $m_i = mf_i m$



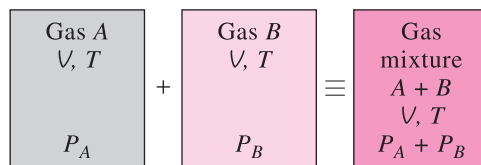
## P-v-T Behaviour of Gas Mixtures

## Theorem (Dalton's Law of Additive Pressures)

The pressure of a gas mixture is equal to the sum of the pressures each gas would exert if it existed alone at the mixture temperature and volume.

$$P = P_1 + P_2 + \dots + P_k = \sum_{i=1}^k P_i(T, V)$$

$P_i$  = partial pressure of component  $i$ .



$$P = P_A + P_B$$

T281

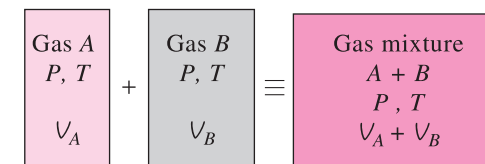


## Amagat's Law of Additive Volumes

The volume of a gas mixture is equal to the sum of the volumes each gas would occupy if it existed alone at the mixture temperature and pressure.

$$V = V_1 + V_2 + \dots + V_k = \sum_{i=1}^k V_i(T, P)$$

$V_i$  = component volume of component  $i$ .



$$V = V_A + V_B$$

$$\Rightarrow \frac{P_i(T, V)}{P} = \frac{n_i R_u T / V}{n R_u T / V} = \frac{n_i}{n} = Y_i : \frac{V_i(T, P)}{V} = \frac{n_i R_u T / P}{n R_u T / P} = \frac{n_i}{n} = Y_i$$

$$\frac{P_i}{P} = \frac{V_i}{V} = \frac{n_i}{n} \equiv Y_i \quad (\text{Ideal gas})$$

T282



**Gibbs-Dalton's Law**

In a mixture of ideal gases each component of the mixture acts as if it were alone in the system at the volume  $V$  and the temperature  $T$  of the mixture.

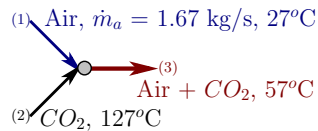
- $U = U_1 + U_2 + \dots + U_k = \sum_{i=1}^k U_i$
- $U = n\tilde{u} = n_1\tilde{u}_1 + n_2\tilde{u}_2 + \dots + n_k\tilde{u}_k = \sum_{i=1}^k n_i\tilde{u}_i$
- $\Rightarrow \tilde{u} = \frac{U}{n} = Y_1\tilde{u}_1 + Y_2\tilde{u}_2 + \dots + Y_k\tilde{u}_k = \sum_{i=1}^k Y_i\tilde{u}_i$
- $U = mu = m_1u_1 + m_2u_2 + \dots + m_ku_k = \sum_{i=1}^k m_iu_i$
- $\Rightarrow u = \frac{U}{m} = mf_1u_1 + mf_2u_2 + \dots + mf_ku_k = \sum_{i=1}^k mf_iu_i$
- $\tilde{u} = \sum Y_i\tilde{u}_i : \tilde{h} = \sum Y_i\tilde{h}_i$
- $u = \sum mf_iu_i : h = \sum mf_ih_i$
- $\tilde{u} \equiv$  specific internal energy on mole basis.
- $u \equiv$  specific internal energy on mass basis.
- $S = S_1 + S_2 + \dots + S_k = \sum_{i=1}^k S_i$



- $\Delta U = \sum_{i=1}^k n_i\tilde{u}_i = \sum_{i=1}^k m_iu_i \Rightarrow \Delta\tilde{u} = \sum_{i=1}^k Y_i\tilde{u}_i ; \Delta u = \sum_{i=1}^k mf_iu_i$
- $\Delta H = \sum_{i=1}^k n_i\tilde{h}_i = \sum_{i=1}^k m_ih_i \Rightarrow \Delta\tilde{h} = \sum_{i=1}^k Y_i\tilde{h}_i ; \Delta h = \sum_{i=1}^k mf_ih_i$
- $\Delta S = \sum_{i=1}^k n_i\tilde{s}_i = \sum_{i=1}^k m_is_i \Rightarrow \Delta\tilde{s} = \sum_{i=1}^k Y_i\tilde{s}_i ; \Delta s = \sum_{i=1}^k mf_is_i$
- $\Delta\tilde{u}_i = \tilde{c}_{v,i}\Delta T, \Delta\tilde{h}_i = \tilde{c}_{p,i}\Delta T, \Delta u_i = c_{v,i}\Delta T, \Delta h_i = c_{p,i}\Delta T$
- $\Delta\tilde{s}_i = \tilde{c}_{p,i} \ln\left(\frac{T_2}{T_1}\right) - R_u \ln\left(\frac{p_{i,2}}{p_{i,1}}\right); \Delta s_i = c_{p,i} \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{p_{i,2}}{p_{i,1}}\right)$
- $p_{i,1} = Y_iP_1$ , partial pressure of component  $i$  at state 1.
- For ideal gas mixtures without change in composition,  $\frac{p_{i,2}}{p_{i,1}} = \frac{P_2}{P_1}$



[Wark Ex. 10.6]: ▷ Adiabatic mixing of two streams at two different temperatures at 1 bar.



- SSSF,  $P_0 = 1$  bar,  $\dot{m}_a = 1.67$  kg/s
- $\dot{m}_1 + \dot{m}_2 - \dot{m}_3 = 0$
- $\dot{m}_1h_1 + \dot{m}_2h_2 - \dot{m}_3h_3 = 0$
- $\dot{m}_a, 1h_{a,1} + \dot{m}_c, 2h_{c,2} = \dot{m}_a, 3h_{a,3} + \dot{m}_c, 3h_{c,3}$

$C_{p,air} = 1005$  J/kgK  
 $C_{p,CO_2} = 1008$  J/kgK

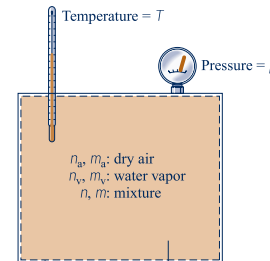
$\Rightarrow \dot{m}_c = 0.712$  kg/s,  $\dot{n}_c = 0.0162$ ,  $\dot{n}_a = 0.0575$  mol/s,  $Y_{a,3} = 0.78$ ,  $Y_c = 0.22$

- $\Delta s_a = 1005 \ln(330/300) - 287 \ln(0.78) = 167$  J/kgK
- $\Delta s_c = 1008 \ln(330/400) - (8314/44) \ln(0.22) = 92$  J/kgK
- $\dot{m}_1s_1 + \dot{m}_2s_2 - \dot{m}_3s_3 + \sum_{j=1}^k \frac{Q_j}{T_j} + \dot{\sigma}_{cv} = 0$
- $\Rightarrow \dot{\sigma}_{cv} = \dot{m}_a\Delta s_a + \dot{m}_c\Delta s_c = 343$  J/K.s



**Moist Air**

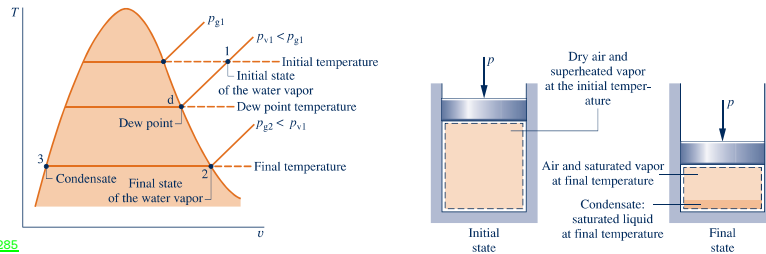
- **Atmospheric air** contains several gaseous components including water vapour and contaminants such as dust and pollutants.
- **Dry air** refers only to the gaseous components when all water vapour and contaminants have been removed.
- **Moist air** refers to a mixture of dry air and water vapour in which the dry air is treated as if it were a pure component.



- $P = P_a + P_v$
- $n = n_a + n_v$
- $m = m_a + m_v$
- $Y_a = \frac{n_a}{n} : Y_v = \frac{n_v}{n}$
- $P_a = Y_aP : P_v = Y_vP$



### Relative Humidity, $\phi$ , Moisture Content, $\omega$

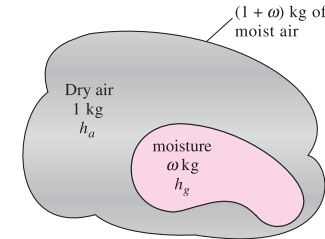


T285

- Relative humidity,  $\phi \equiv \frac{P_v}{P_g} \Big|_{T,P} : P_g = P_{sat}@T$
  - Moisture content,  $\omega \equiv \frac{m_v}{m_a}$
- $\rightarrow \omega = \frac{m_v}{m_a} = \frac{P_v VM_v / R_u T}{P_a VM_a / R_u T} = \frac{M_v P_v}{M_a P_a} \simeq 0.622 \frac{P_v}{P_a} : 18.0/28.95 = 0.622.$

$$\omega = 0.622 \frac{P_v}{P_a} = 0.622 \frac{P_v}{P - P_v} = 0.622 \frac{\phi P_g}{P - \phi P_g}$$

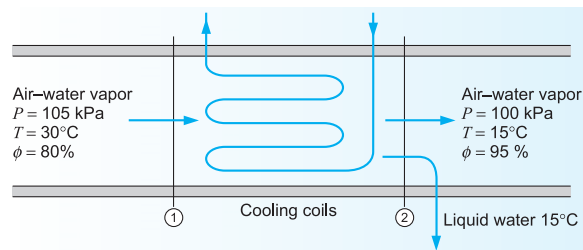
### Moist Air Enthalpy, $h$



T283

- $h = h_a + \omega h_g$  kJ/kg dry air
- $H = H_a + H_v = m_a h_a + m_v H_v$
  - $h \equiv \frac{H}{m_a} = h_a + \frac{m_v}{m_a} h_v = h_a + \omega h_v$
  - $h_v \simeq h_g(T) \Rightarrow \boxed{h = h_a + \omega h_g}$
  - $h_a = c_{pa} T = 1.005 T$  [kJ/kg da]
  - $h_w = c_{pw} T = 4.1867 T$  [kJ/kg water]
  - $h_g = 2501.7 + 1.82 T$  [kJ/kg water vapour]
- $\Leftarrow T$  in  $^{\circ}\text{C}$ .

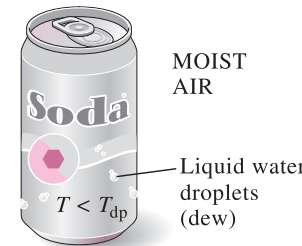
[Borgnakke Ex. 11.5]:  $\triangleright$  Cooling and dehumidification in a cooling coil of an Air-Conditioner.



T295

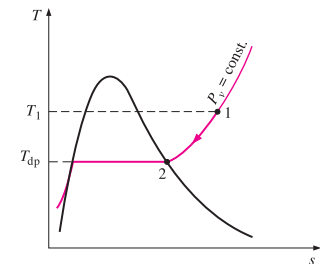
- Mass balance:  $\dot{m}_{a1} = \dot{m}_{a2} = \dot{m}_a : \dot{m}_{v1} = \dot{m}_{v2} + \dot{m}_{f2}$
  - Energy balance:  $\dot{Q}_{cv} + \sum \dot{m}_i h_i = \sum \dot{m}_e h_e$
  - $\phi = \frac{P_v}{P_g} : \omega = 0.622 \frac{\phi P_g}{P - \phi P_g}$
  - $h_a = 1.005 T : h_v = h_g = 2501.7 + 1.82 T : h_w = 4.186 T$
- $\Rightarrow \frac{\dot{Q}_{cv}}{\dot{m}_a} = (h_{a2} + \omega_2 h_{v2}) - (h_{a1} + \omega_1 h_{v1}) + (\omega_1 - \omega_2) h_{w2} = -41.64$  kJ/kg da

### Dew-point Temperature, $T_{dp}$



T286

When the temperature of a cold drink is below the  $T_{dp}$  of the surrounding air, it sweats.

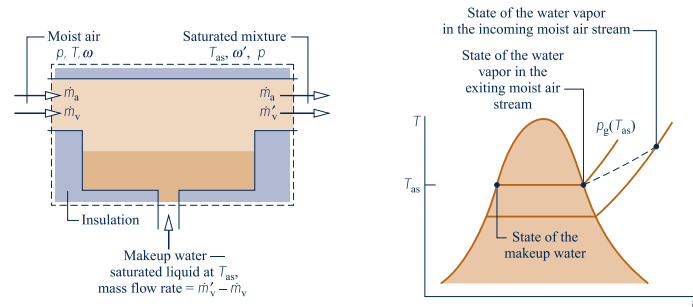


T287

Constant-pressure cooling of moist air and the dew-point temperature on the  $T$ - $s$  diagram of water.

$$T_{dp} = T_{sat}(P_v)$$

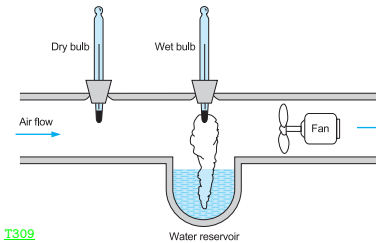
## Adiabatic Saturation Process



T288

- $h_v(T) \simeq h_g(T)$ ,  $h_v(T_{as}) \simeq h_g(T_{as})$ ,  $\omega = m_v/m_a$ ,  $\omega' = m'_v/m_a$
- $m_a h_a(T) + m_v h_g(T) + (m'_v - m_v) h_w(T_{as}) = m_a h_a(T_{as}) + m'_v h_g(T_{as})$

$$\omega = \frac{h_a(T_{as}) - h_a(T) + \omega' [h_g(T_{as}) - h_f(T_{as})]}{h_g(T) - h_f(T_{as})} : \omega' = 0.622 \frac{p_g(T_{as})}{P - p_g(T_{as})}$$

Wet Bulb Temperature,  $T_{wb}$  and Psychrometer

T309

*Adiabatic saturation process provides a mean to measure humidity content of moist air, and the process can be approximated by using a wet-bulb thermometer.*

- **Wet-bulb temperature,  $T_{wb}$**  is read from a wet-bulb thermometer, which is an ordinary liquid-in-glass thermometer whose bulb is enclosed by a wick moistened with water.
- **Dry-bulb temperature,  $T_{db}$**  refers simply to the temperature that would be measured by a thermometer placed in the mixture. Often a wet-bulb thermometer is mounted together with a dry-bulb thermometer to form an instrument called a **psychrometer**.

Estimation of  $P_{sat}$  and  $\phi$ 

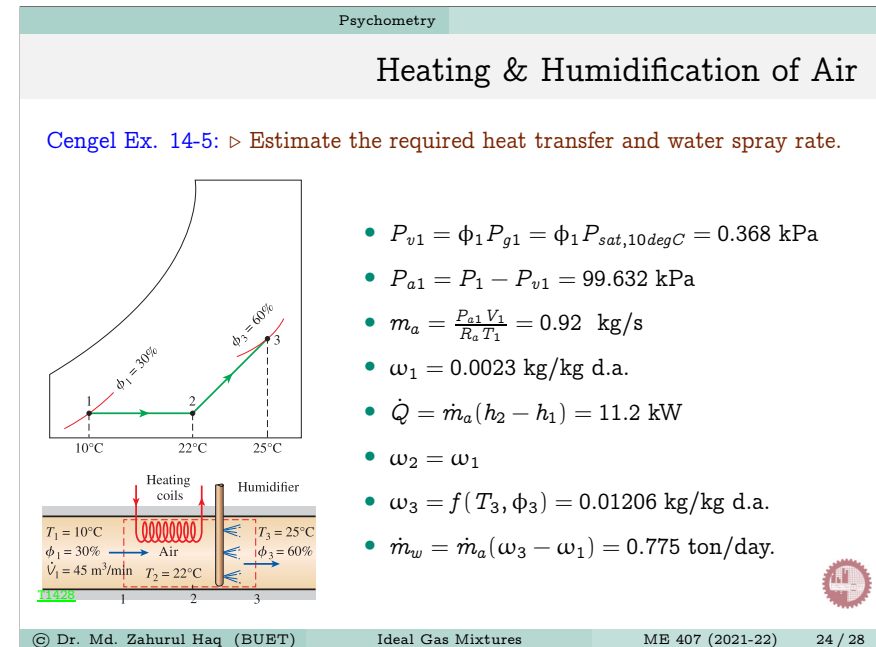
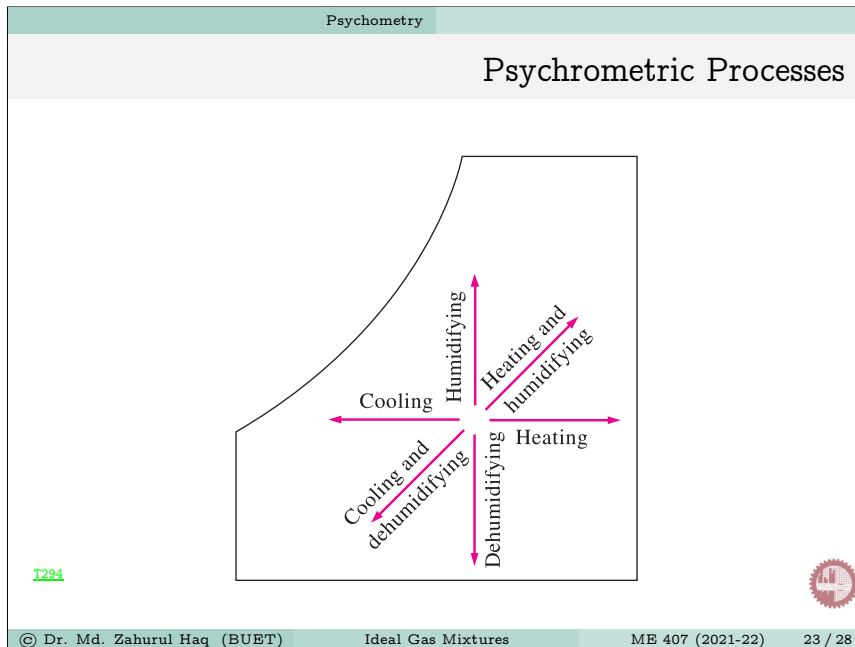
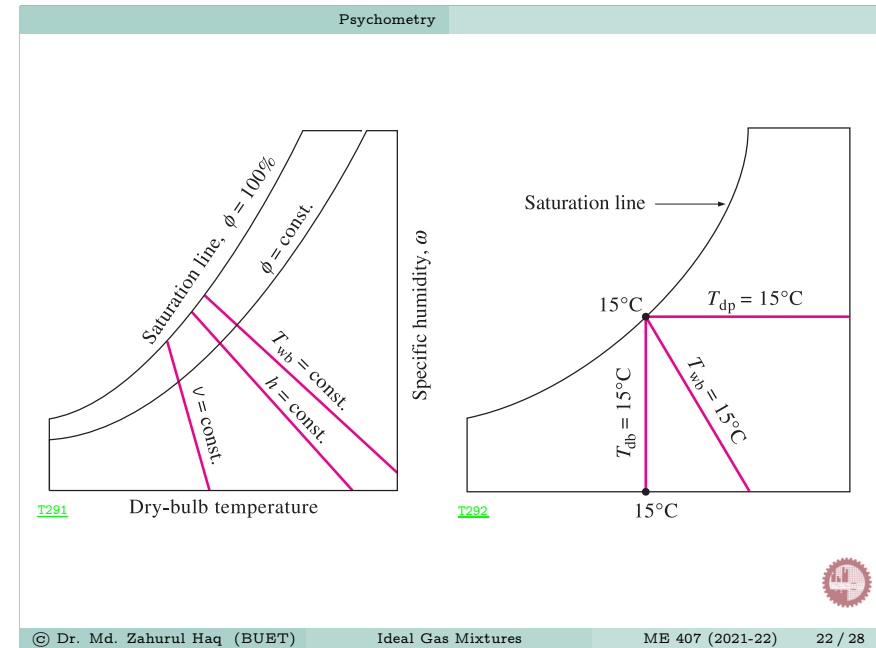
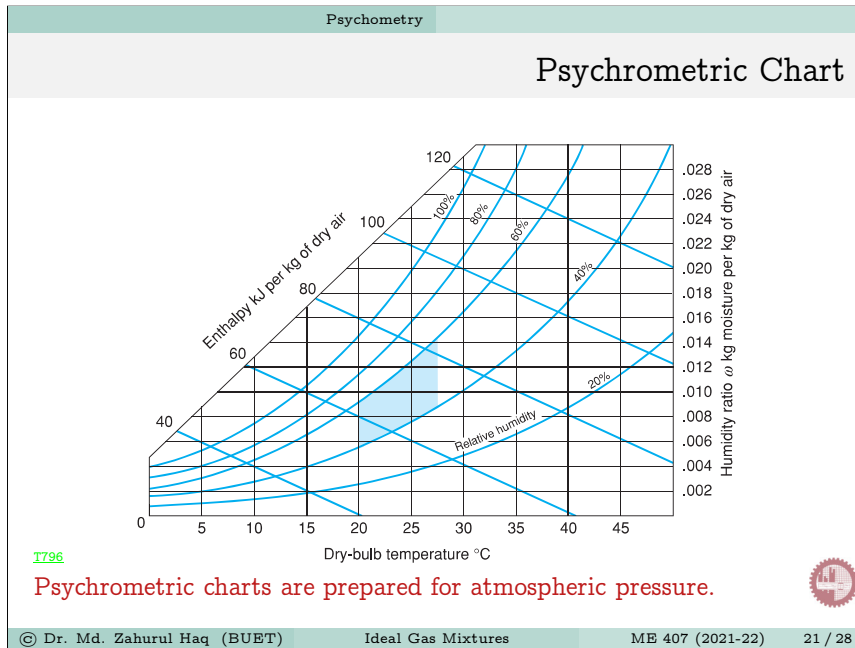
- $P_{sat}(T) = P_c \times 10^{k(1 - T_c/T)} = 22.10 \times 10^{k(1 - 647.3/T)}$  (MPa)
- $k = 4.39553 - 6.2442 \left(\frac{T}{1000}\right) + 9.953 \left(\frac{T}{1000}\right)^2 - 5.151 \left(\frac{T}{1000}\right)^3$
- $\phi = \frac{P_{sat}(T_{wb}) - P_m}{P_{sat}(T_{db})}$
- $P_m = P \left( \frac{T_{db} - T_{wb}}{1514} \right) \left( 1 + \frac{T_{wb} - 273.2}{873} \right)$
- $P_{sat}(T_{wb}) \equiv$  saturation pressure corresponding to  $T_{wb}$
- $P_{sat}(T_{db}) \equiv$  saturation pressure corresponding to  $T_{db}$
- $P_m \equiv$  partial pressure of water vapour due to depression of w.b.t. below d.b.t.



**Example:**  $\triangleright T_{db} = 25^\circ C$  &  $T_{wb} = 20^\circ C$ , without using psychrometric chart:

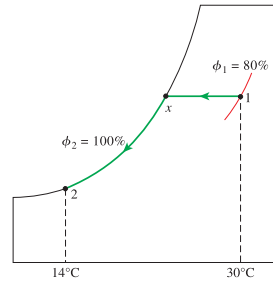
- $P_g(T) = P_{sat}@T \Rightarrow P_g(T_{db}) = 3.169$  kPa,  $P_g(T_{wb}) = 2.339$  kPa
- $\phi' = 1.0$  &  $\omega' = 0.622 \frac{\phi_2 P_g(T_{wb})}{P - \phi_2 P_g(T_{wb})} \Rightarrow \omega' = 0.0147$
- $h_a(T_{wb}) - h_a(T_{db}) = 1.005(T_{wb} - T_{db}) : h_w(T_{wb}) = 4.186 T_{wb}$
- $h_g(T_{db}) = 2547.2$  kJ/kg,  $h_g(T_{wb}) = 2538.1$  kJ/kg
- $\omega = \frac{h_a(T_{wb}) - h_a(T_{db}) + \omega' [h_g(T_{wb}) - h_w(T_{wb})]}{h_g(T_{db}) - h_f(T_{wb})} = 0.0126 \blacktriangleleft$
- $\omega = 0.622 \frac{\phi P_g(T_{db})}{P - \phi P_g(T_{db})} \Rightarrow \phi = 0.635 \blacktriangleleft$
- $h = h_a(T_{db}) + \omega h_g(T_{db}) = 57.23$  kJ/kgda  $\blacktriangleleft$
- $P_v = \phi P_g(T_{db}) = 2012.5$  kPa
- $T_{dp} = T_{sat}@P_v = 17.59^\circ C \blacktriangleleft$



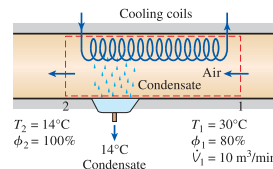


## Cooling &amp; Dehumidification of Air

Cengel Ex. 14-6: ▷ Estimate the required rate of heat and moisture removal.



- $\dot{m}_w = 188.6 \text{ kg/day}$
- $\dot{Q} = 8.5 \text{ kW} \approx 2.5 \text{ RT.}$



$T_2 = 14^\circ\text{C}$   
 $\phi_2 = 100\%$   
 $T_1 = 30^\circ\text{C}$   
 $\phi_1 = 80\%$   
 $V_1 = 10 \text{ m}^3/\text{min}$

T1429

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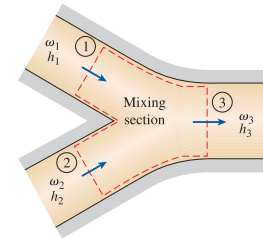
Ideal Gas Mixtures

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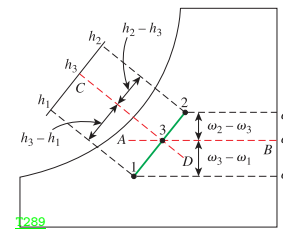
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## Adiabatic Mixing of Two Moist Air Streams



- Dry air :  $\dot{m}_{a1} + \dot{m}_{a2} = \dot{m}_{a3}$
  - Water vapour :  $\dot{m}_{v1} + \dot{m}_{v2} = \dot{m}_{v3}$
- $$\Rightarrow \omega_1 \dot{m}_{a1} + \omega_2 \dot{m}_{a2} = \omega_3 \dot{m}_{a3}$$
- $\dot{m}_{a1}(h_{a1} + \omega_1 h_{v1}) + \dot{m}_{a2}(h_{a2} + \omega_2 h_{v2}) = \dot{m}_{a3}(h_{a3} + \omega_3 h_{v3})$
- $$\Rightarrow \dot{m}_{a1} h_1 + \dot{m}_{a2} h_2 = \dot{m}_{a3} h_3$$
- $$\Rightarrow \frac{\dot{m}_{a1}}{\dot{m}_{a2}} = \frac{\omega_2 - \omega_3}{\omega_3 - \omega_1} = \frac{h_2 - h_3}{h_3 - h_1}$$



T289

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Ideal Gas Mixtures

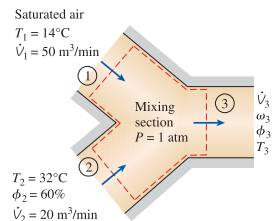
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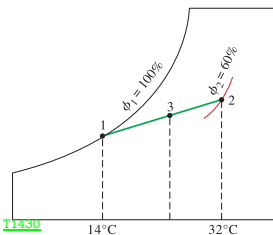


## Mixing of Conditioned Air with Outdoor Air

Cengel Ex. 14-8: ▷ Estimate state 3.



Saturated air  
 $T_1 = 14^\circ\text{C}$   
 $V_1 = 50 \text{ m}^3/\text{min}$



- $T_3 = 19^\circ\text{C}$
- $\phi_3 = 89\%$

T1430

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Condensation at Air-Compressor: ▷ Air at  $30^\circ\text{C}$  and 80% RH is isothermally compressed to 1.0 MPa. Estimate the psychrometric condition of the compressed air. If same atmospheric air is expanded to 0.01 MPa, estimate the psychrometric condition. Try to solve the problem using psychrometric chart.

a : Compression process:

- $P_{sat} = 0.00425 \text{ MPa}$ ,  $\omega_1 = 0.02166 \text{ kg/kg d.a.}$
- If  $\omega_2 = \omega_1 \rightarrow \phi_2 = 7.88 \gg 1.0$
- So, water condensation occurs, and final state is  $\phi_2 = 100\%$  at  $30^\circ\text{C}$ .
- $\omega_2 = 0.00265 \text{ kg/kg d.a.}$
- $\Delta\omega = 0.019 \text{ kg/kg d.a.}$

b : Expansion process:

- $\omega_2 = \omega_1 = 0.02166 \text{ kg/kg d.a.}$
- $\phi_2 = 79\%$ .

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