

Exergy Analysis of Engineering Applications

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ME 407: Advanced Thermodynamics
<http://zahurul.buet.ac.bd/ME407/>



Equations: CM & CV Systems

CM System:

- $Q - W = \Delta U$
- $W_u = W - P_0 \Delta V = W - W_0$
- $\phi = (u - u_0) + P_0(v - v_0) - T_0(s - s_0)$
- $\Phi_Q = \sum_{j=1}^n Q_j \left(1 - \frac{T_0}{T_j}\right)$
- $\Delta\Phi = \Phi_Q - W_u - I_{cm}$

SSSF CV System:

- $Q - W_{sf} = m(\Delta h + \Delta pe + \Delta he) = m\Delta h$
- $W_u = W_{sf} - P_0 \Delta V^0 = W_{sf}$
- $\psi = (h - h_0) - T_0(s - s_0) + \frac{V^2}{2} + gz$
- $\Phi_Q = \sum_{j=1}^n Q_j \left(1 - \frac{T_0}{T_j}\right)$
- $\Delta(m\psi) = \sum_e \dot{m}_e \psi_e - \sum_i \dot{m}_i \psi_i = \dot{\Phi}_Q - \dot{W}_u - I_{cv}$



Second Law Efficiency (η_{II}) or Effectiveness (ϵ)

- A performance parameter based on the exergy concept is known as Second Law Efficiency (η_{II}) or as Second Law Effectiveness (ϵ).
- A **first-law efficiency** gauges how well the energy is used when compared against an ideal process, whereas an **effectiveness** indicates how well exergy is utilized.
- Second Law efficiency can approach unity for a thermodynamically perfect device and it therefore provides a true indication of the efficiency of the system.

$$\eta_{II} \equiv \epsilon \equiv \frac{\text{exergy of desired output}}{\text{exergy supplied}} = 1 - \frac{\text{exergy destruction}}{\text{exergy supplied}}$$



Adiabatic Compression & Pumping

CM process:

- $\Delta\Phi = \Phi_Q - W_u - I_{cm}$
- $W_u = W_{act} + P_0 \Delta V$
- $\epsilon \equiv \frac{\Delta\Phi}{W_{act}}$

CV process:

- $\Delta\Psi = \Phi_Q - W_u - I_{cv}$
- $W_u = W_{act}$
- $\epsilon \equiv \frac{\Delta\Psi}{W_{act}}$

- Effectiveness (ϵ) is defined as the increase in the specific availability of the fluid per unit of actual work input.
- First law efficiency, $\eta \equiv \frac{W_s}{W_{act}}$.



Exergy Analysis

Steam/Gas Turbine, Throttling & Nozzle

Turbines:

- $\eta = \frac{w_a}{\Delta h}$
- $\Delta \psi = \phi_Q - w_u - i_{cv}$
- $\epsilon = \frac{w_a}{\Delta \psi}$

Throttling:

- $0 = q^0 - w^0 + h_i - h_e - \Delta ke^0$
- $\Delta \psi = \phi_Q^0 - w_u^0 - I_{cv}$
- $\epsilon = \frac{\psi_e}{\psi_i}$

Nozzle:

- $q^0 + w^0 = h_e - h_i + \Delta ke$
- $\eta = \frac{\Delta ke_a}{\Delta ke_s}$
- $\Delta \psi = \Phi_Q^0 + w_u^0 - I_{cv}$
- $\epsilon = \frac{w_e}{\Psi_i}$

T475

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Exergy Analysis

Cyclic Heat Engine

Cyclic Heat Engine:

- $\eta_{th} = \frac{W_{out}}{Q_H} = \frac{180}{500} = 0.36$
- $\eta_{Carnot} = 1 - \frac{T_L}{T_H} = 1 - \frac{300}{1200} = 0.75$
- $\Phi_Q = Q_H \eta_{Carnot} = (500)(0.75) = 375 \text{ kW}$
- $\epsilon = \frac{W_{out}}{\Phi_Q} = \frac{180}{375} = 0.48$
- $\epsilon = \frac{W_{out}}{\Phi_Q} = \frac{\eta_{th} Q_H}{\eta_{Carnot} Q_H} = \frac{\eta_{th}}{\eta_{Carnot}} = \frac{0.36}{0.75} = 0.48$

T483

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Exergy Analysis

Power Plant Using Different Heat Sources

Power plant:

- $\dot{Q}_{HT} = 25 \text{ kW}$, $T_{HT} = 825^\circ\text{C}$
- $\dot{Q}_{MT} = 50 \text{ kW}$, $T_{MT} = 240^\circ\text{C}$
- $\dot{W}_{out} = 12 \text{ kW}$
- \dot{Q}_0 is rejected to $T_0 = 20^\circ\text{C}$

Energy balance: $\dot{Q}_{HT} + \dot{Q}_{MT} - \dot{Q}_{0,rev} - \dot{W}_{out,rev} = 0$

Entropy balance: $\frac{\dot{Q}_{HT}}{T_{HT}} + \frac{\dot{Q}_{MT}}{T_{MT}} - \frac{\dot{Q}_{0,rev}}{T_0} = 0$

Efficiencies:

- $\eta_I = \frac{W_{out}}{\dot{Q}_{HT} + \dot{Q}_{MT}} = \frac{12}{25+50} = 16\%$
- $\Phi_{Q,HT} = 25 \left(1 - \frac{293}{1098}\right) = 18.33$
- $\Phi_{Q,MT} = 50 \left(1 - \frac{293}{513}\right) = 21.44$
- $\eta_{II} = \frac{W_{out}}{\Phi_{Q,HT} + \Phi_{Q,MT}} = 30.2\%$

T349

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Applications of Exergy Based Analysis

Heat Conduction Through Wall

Moran, Ex. 7-3 ▷

Wall Parameters:

- Insulation: $k = 0.05 \times 10^{-3} \text{ kW/m}\cdot\text{K}$
- Area: A
- Thickness: $L = 0.066 \text{ m}$
- Left Boundary: $T_1 = 575 \text{ K}$
- Right Boundary: $T_2 = 310 \text{ K}$
- Surroundings: 293 K

Equations:

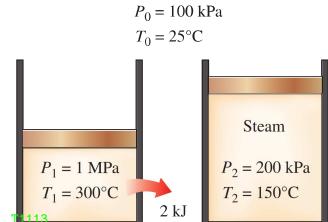
- $\dot{q} = -k \left[\frac{T_2 - T_1}{L} \right] = 0.2 \frac{\text{kW}}{\text{m}^2}$
- $\dot{\Phi}_{Q,in} = q \left[1 - \frac{T_0}{T_1} \right] = 0.1 \frac{\text{kW}}{\text{m}^2}$
- $\dot{\Phi}_{Q,out} = q \left[1 - \frac{T_0}{T_2} \right] = 0.01 \frac{\text{kW}}{\text{m}^2}$
- $I_Q = \dot{\Phi}_{Q,in} - \dot{\Phi}_{Q,out} = 0.09 \frac{\text{kW}}{\text{m}^2}$

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CM System: Expansion of Steam inside Cylinder

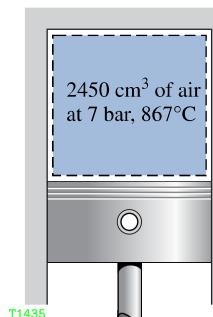
Cengel, Ex. 8-11 ▷ Piston-cylinder assembly contains 0.05 kg steam.



- $Q - W = m(\Delta u + \Delta KE + \Delta PE) \simeq m\Delta u$
- $\phi \equiv (u - u_0) + P_0(v - v_0) - T_0(s - s_0)$
- $W_u = W - W_0 = 8.826 - 3.509 = 5.317 \text{ kJ}$
- $\phi_Q = 0$: Heat loss to T_0
- $\Delta\phi = -9.648 \text{ kJ}$
- $I_{CM} = \phi_Q - W_u - \Delta\phi = 4.331 \text{ kJ}$
- $\epsilon = \frac{W_u}{-\Delta\phi} = 0.551$

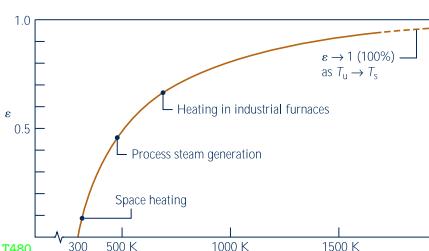
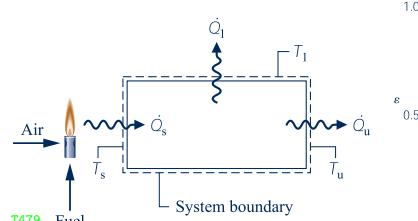
Evaluating the Exergy of Exhaust Gas

Moran Ex. 7.1 ▷ A cylinder of an internal combustion engine contains 2450 cm^3 of gaseous combustion products at a pressure of 7 bar and a temperature of 867°C just before the exhaust valve opens. Determine the specific exergy of the gas, in kJ/kg . Assume, the combustion products as air as ideal gas.



- $T_0 = 300 \text{ K}, P_0 = 1.0 \text{ bar}$
- $\phi \equiv (u - u_0) + P_0(v - v_0) - T_0(s - s_0)$
- $u - u_0 = c_v(T - T_0) = 600 \text{ kJ/kg}$
- $P_0(v - v_0) = R(\frac{P_0 T}{P} - T_0) = 39.36 \text{ kJ/kg}$
- $s - s_0 = c_p \ln(T/T_0) - R \ln(P/P_0) = 0.7870 \text{ kJ/kg}$
- $\phi = 324.54 \text{ kJ/kg}$
- If accurate thermodynamic properties are used, $\phi = 369.45 \text{ kJ/kg}$

Efficient Use of Heat Source



- First Law: $\frac{dE}{dt} = (\dot{Q}_s - \dot{Q}_u - \dot{Q}_i) - \dot{W}$
- $\eta = \frac{\dot{Q}_u}{\dot{Q}_s}$: $\dot{Q}_s \equiv$ heat transfer from source, $\dot{Q}_u \equiv$ useful heat transfer
- SSSF: $\Delta\Psi^0 = \Delta\Phi_Q - \dot{W}_{act}^0 - I_{cm} = \Phi_{Q,s} - \Phi_{Q,u} - \Phi_{Q,1} - I_{cm}$

$$\Rightarrow \epsilon \equiv \frac{\Phi_{Q,u}}{\Phi_{Q,s}} = \frac{\dot{Q}_u(1-T_0/T_u)}{\dot{Q}_s(1-T_0/T_s)} = \eta \left[\frac{1-T_0/T_u}{1-T_0/T_s} \right]$$

Wark (1995), Ex. 3.2: ▷ A heat transfer of 100 kJ occurs between reservoirs of 1000 K and 300 K. Estimate the effectiveness of the case and also for reservoir temperature of 40°C, 200°C & 420°C. $T_0 = 300 \text{ K}$.

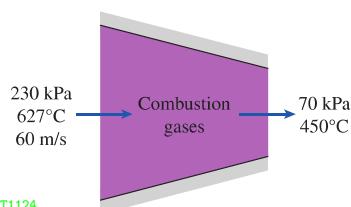
$$\epsilon \equiv \frac{\Phi_{Q,u}}{\Phi_{Q,s}} = \frac{\dot{Q}_u(1-T_0/T_u)}{\dot{Q}_s(1-T_0/T_s)} = \eta \left[\frac{1-T_0/T_u}{1-T_0/T_s} \right]$$

- For all cases, $\Phi_{Q,s} = \dot{Q}_s(1 - T_0/T_s) = 100(1 - 300/1000) = 70 \text{ kJ}$
- For sink temperature of 420°C (e.g. furnace operation), $\Phi_{Q,u} = \dot{Q}_s(1 - T_0/T_u) = 100(1 - 300/673) = 56.7 \text{ kJ}$.
- $I_{cm} = \Phi_{Q,s} - \Phi_{Q,u} = 13.3 \text{ kJ}$
- $\epsilon = \frac{\Phi_{Q,u}}{\Phi_{Q,s}} = 0.81$

use	$T_u [\text{K}]$	$\Phi_U [\text{kJ}]$	$I_Q [\text{kJ}]$	ϵ
Atmospheric sink	300	0	70	0.00
Space heating	313	4.2	65.8	0.06
Process steam generation	473	36.6	33.4	0.52
Furnace operation	693	56.7	13.3	0.81

Nozzle

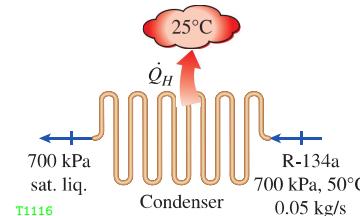
Cengel, P. 8-71 ▷ Hot combustion gases enter the nozzle of a turbojet engine. Assuming the nozzle to be adiabatic and the surroundings to be at 20°C, determine (a) the exit velocity and (b) the decrease in the exergy of the gases. Take air properties for the combustion gases.



- $V_2 = \sqrt{V_1^2 - \Delta KE} = 627 \text{ m/s}$
- $\psi_1 = 368.9 \text{ kJ}$
- $\psi_2 = 339.4 \text{ kJ}$
- $I_{cv} - \Delta\psi = 29.5 \text{ kJ}$
- $\epsilon = \frac{\psi_2}{\psi_1} = 0.92$

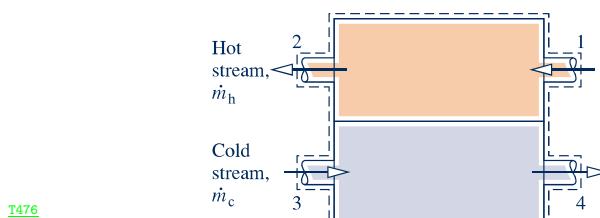
Air Cooled Condenser

Cengel, P. 8-63 ▷ Determine (a) the rate of heat rejected in the condenser, (b) the COP of this refrigeration cycle if the cooling load at these conditions is 6 kW, and (c) the rate of exergy destruction in the condenser.



- $Q_L = 6 \text{ kW}$
- $Q_H = \dot{m}(h_2 - h_1) = 9.98 \text{ kW}$
- $COP = \frac{Q_L}{W_{in}} = \frac{Q_L}{Q_H - Q_L} = 1.5$
- $I_{cv} = -\Delta\psi = 0.0998 \text{ kW}$
- $\epsilon = \frac{\psi_2}{\psi_1} = 0.955$

Heat Exchange without Mixing



- $w = 0, q = 0, \Delta ke = 0, \Delta pe = 0$
- SSSF Energy:** $0 = 0 - 0 + \sum_i (mh)_i - \sum_e (mh)_e$
- $\Rightarrow m_1 h_1 + m_3 h_3 = m_1 h_2 + m_4 h_4 \rightarrow [m_c(h_4 - h_3) = -m_h(h_2 - h_1)]$

Exergy balance:

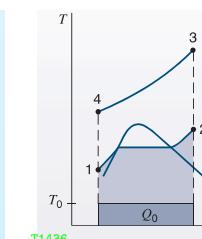
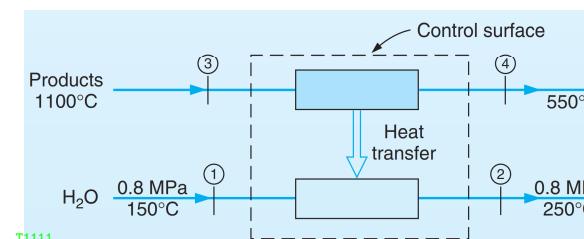
$$\Delta(m\Psi) = \Phi_Q^0 - W^0 - I_{cv} \rightarrow m_c(\psi_4 - \psi_3) + m_h(\psi_2 - \psi_1) = -I_{cv}$$

$$\Rightarrow \boxed{\epsilon \equiv \frac{m_c(\psi_4 - \psi_3)}{-m_h(\psi_2 - \psi_1)}} \quad \text{or} \quad \epsilon \equiv \frac{m_c\psi_4 + m_h\psi_2}{m_c\psi_3 + m_h\psi_1}$$

- The first form of ϵ for heat exchanger is usually preferred.

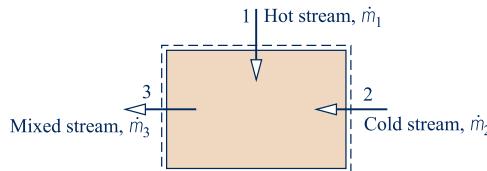
Boiler

Borgnakke, Ex. 8.6: ▷ Determine the second-law efficiency for this process and the irreversibility per kilogram of water evaporated. Assume, c_p of the products of combustion is 1.155 kJ/kg K.



- $\frac{m_{gas}}{m_{water}} = \left[\frac{h_2 - h_1}{h_3 - h_4} \right] = 3.685 \text{ kg/kg}$
- $\epsilon = \frac{m_{gas}(\psi_2 - \psi_1)}{m_{water}(\psi_3 - \psi_4)} = 0.458$
- $I_{cv} = -\Delta\psi = 9.09 \text{ kW}$

Heat Exchange with Mixing

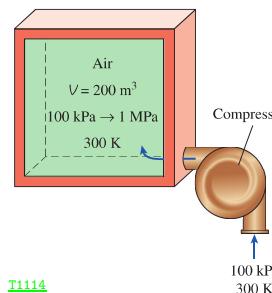


T481

- $w = 0, q = 0, \Delta ke = 0, \Delta pe = 0$
 - SSSF Energy:** $0 = 0 - 0 + \sum_i (mh)_i - \sum_e (mh)_e$
- $$\Rightarrow m_1 h_1 + m_2 h_2 = m_3 h_3 \rightarrow [m_c(h_3 - h_2)] = -m_h(h_3 - h_1)$$
- Exergy balance:**
- $$\Delta(m\Psi) = \Phi_Q^0 - W^0 - Icv \rightarrow m_c(\psi_3 - \psi_2) + m_h(\psi_3 - \psi_1) = -I_{cv}$$
- $$\Rightarrow \boxed{\epsilon \equiv \frac{m_c(\psi_3 - \psi_2)}{-m_h(\psi_3 - \psi_1)} \quad \text{or} \quad \epsilon \equiv \frac{(m_c + m_h)\psi_3}{m_c\psi_2 + m_h\psi_1}}$$

Charging a Compressed Air System

Cengel Ex. 8-17 ▷ A 200-m³ rigid tank initially contains atmospheric air at 100 kPa and 300 K and is to be used as a storage vessel for compressed air at 1 MPa and 300 K. Determine the minimum work requirement for this process.

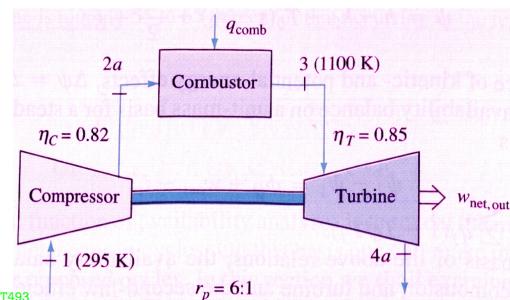


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- $\phi = \psi = 0$, air within tank and supplied are at ambient condition.
- $m_2 = \frac{P_2 V}{R T_2} = 2323 \text{ kg}$
- $\phi = (u - u_0) + P_0(v - v_0) - T_0(s - s_0)$
- $u - u_0 = 0$, as $T_2 = T_0$
- $s - s_0 = c_p \ln(T_2/T_0) - R \ln(P_2/P_0) = -R \ln(P_2/P_0)$
- $P_0(v - v_0) = R T_0 (P_0/P_2 - 1)$
- $\phi_2 = 120.78 \text{ kJ/kg}$
- Minimum work required = $m_2 \phi_2 = 281 \text{ MJ}$

Gas Turbine Cycle Analysis

Wark (1999), Ex.15.12: ▷



T493

- Energy:** $q - w = \Delta h + \Delta e^0 + \Delta p^0$
- Exergy:** $\Delta \psi = \Phi_Q - w_u - Icv$
- $\eta_C = \frac{w_{c,s}}{w_{c,a}} = \frac{h_{2s} - h_1}{h_{2a} - h_1}$
- $\eta_t = \frac{w_{t,a}}{w_{t,s}} = \frac{h_{4a} - h_3}{h_{4s} - h_3}$

- Combustor:** $w = 0, I_{c,comb} = 0$ (generally assumed).
- $\Rightarrow q_{comb} = h_3 - h_{2a} = 630 \text{ kJ/kg}, \Phi_Q = \psi_3 - \psi_{2a} = 391 \text{ kJ/kg}$
- Compressor:**

 - $q = 0, w_{c,s} = (h_1 - h_{2s}), w_{c,a} = w_{c,s}/\eta_c = (h_1 - h_{2a})$
 - $\Rightarrow w_{c,a} = -242 \text{ kJ/kg}, I_{c,comp} = -w_{c,a} - (\psi_{2a} - \psi_1) = 25.5 \text{ kJ/kg}$

- Turbine:** $q = 0, w_{t,s} = (h_3 - h_{4s}), w_{t,a} = \eta_t w_{t,s} = (h_3 - h_{4a})$
- $\Rightarrow w_{t,a} = 388 \text{ kJ/kg}, I_{c,turb} = -w_{t,a} - (\psi_{4a} - \psi_3) = 28.1 \text{ kJ/kg}$
- $\Rightarrow w_{net} = w_{t,a} + w_{c,a} = 146.1 \text{ kJ/kg}$
- $\Rightarrow \eta = \frac{w_{net}}{q_{comb}} = 0.232 \blacktriangleleft$
- $\Rightarrow \epsilon = \frac{w_{net}}{\Phi_Q} = 0.373 \blacktriangleleft$
- Energy balance:** $q - w = 483.7, h_{4a} - h_1 = 483.7 \text{ kJ/kg}$
- Exergy balance:** $\psi_{4a} - \psi_1 = 191.5, \Phi_Q - w_{net} - I_{c,total} = 191.5 \text{ kJ/kg}$

Engineering Cycles: Energy & Exergy Audit	Gas Turbine Cycle
Energy Analysis:	
w_ca	= -2.4208E+02 [kJ/kg]
w_ta	= 3.8819E+02 [kJ/kg]
w_net	= 1.4611E+02 [kJ/kg]
q_in_comb	= 6.2985E+02 [kJ/kg]
e_in	= 8.6874E+02 [kJ/kg]
e_out	= 8.6874E+02 [kJ/kg]
eta	= 2.3198E-01 [-]
Exergy Analysis:	
compressor	= 2.1661E+02 [kJ/kg]
combustor	= 3.9124E+02 [kJ/kg]
turbine	= -4.1633E+02 [kJ/kg]
incoming fluid	= -1.1141E+00 [kJ/kg]
outgoing fluid	= 1.9040E+02 [kJ/kg]
sum d_psi	= 0.0000E+00 [kJ/kg]
i_c	= 2.5466E+01 [kJ/kg]
i_t	= 2.8143E+01 [kJ/kg]
i	= 5.3609E+01 [kJ/kg]
epsilon	= 3.7346E-01 [-]

Engineering Cycles: Energy & Exergy Audit

Rankine Cycle

Simple Steam Power Cycle Analysis

Wark (1999), Ex.16.12: ▷

30 bars
3 500°C
 $\eta_T = 0.82$
Superheater
Turbine $w_{T,\text{out}}$
Boiler
Condenser
 $T_{\text{cw,in}}$
 $T_{\text{cw,out}}$
 q_{out}
 0.1 bar
 1
 $w_{P,\text{in}}$
 $\eta_P = 0.78$

T494

- **Energy:**

$$q - w = \Delta h + \Delta ke^0 + \Delta pe^0$$

- **Exergy:**

$$\Delta \Psi = \Phi_Q - w_u - I cv$$

- $\eta_p = \frac{w_{p,s}}{w_{p,a}} = \frac{h_{2s} - h_1}{h_{2a} - h_1}$

- $\eta_t = \frac{w_{t,a}}{w_{t,s}} = \frac{h_{4a} - h_3}{h_{4s} - h_3}$

- $w_{p,s} = (h_1 - h_{2,s}) = \frac{P_1 - P_2}{\rho_1}$

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Exergy Applications

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26 / 31

Engineering Cycles: Energy & Exergy Audit Rankine Cycle

- **pump:** $q = 0$, $w_{p,a} = w_{p,s}/\eta_p$
- $\Rightarrow w_{p,a} = -3.87 \text{ kJ/kg}$, $I_{c,pump} = -w_{p,a} - (\Psi_{2,a} - \Psi_1) = 0.80 \text{ kJ/kg}$
- **Boiler:** $w = 0$, $I_{c,comb} = 0$ (generally assumed).
- $\Rightarrow q_{comb} = h_3 - h_{2a} = 3262.0 \text{ kJ/kg}$, $\Phi_Q = \Psi_3 - \Psi_{2a} = 1300.0 \text{ kJ/kg}$
- **Turbine:** $q = 0$, $w_{t,s} = (h_3 - h_{4,s})$, $w_{t,a} = \eta_t w_{t,s} = (h_3 - h_{4,a})$
- $\Rightarrow w_{t,a} = 954.9 \text{ kJ/kg}$, $I_{c,turb} = -w_{t,a} - (\Psi_{4,a} - \Psi_3) = 190.5 \text{ kJ/kg}$
- $\Rightarrow w_{net} = w_{t,a} + w_{c,a} = 951.1 \text{ kJ/kg}$
- $\Rightarrow \eta = \frac{w_{net,out}}{q_{comb}} = 0.291 \blacktriangleleft$
- $\Rightarrow \epsilon = \frac{w_{net,out}}{\Phi_Q} = 0.731 \blacktriangleleft$
- **Condenser:**
- $q_{cond} = h_1 - h_{4,a} = -2310$, $\psi_{cond} = \Psi_1 - \Psi_{4,a} = -157.1 \text{ kJ/kg}$
- **Energy balance:** $q - w = 3200 - 2310 - 951 = 0$
- **Exergy balance:** $\phi_Q - w_{net} - I_{c,total} = 1300 - 157 - 951 - 191 = 0$

Engineering Cycles: Energy & Exergy Audit	Rankine Cycle
Energy Analysis:	
Pump	= -3.870E+00 [kJ/kg]
Boiler	= 3.262E+03 [kJ/kg]
Turbine	= 9.549E+02 [kJ/kg]
Condenser	= -2.310E+03 [kJ/kg]
Net heat	= 9.511E+02 [kJ/kg]
Net work	= 9.511E+02 [kJ/kg]
Net energy in	= 3.265E+03 [kJ/kg]
Net energy out	= 3.265E+03 [kJ/kg]
eta	= 2.916E-01 [-]
Exergy Analysis:	
Pump	= 3.075E+00 [kJ/kg]
Boiler	= 1.299E+03 [kJ/kg]
Turbine	= -1.151E+03 [kJ/kg]
Condenser	= -1.507E+02 [kJ/kg]
Sum of d_psi	= 5.821E-14 [kJ/kg]
i_p	= 7.954E-01 [kJ/kg]
i_t	= 1.959E+02 [kJ/kg]
i	= 1.967E+02 [kJ/kg]
epsilon	= 7.324E-01 [-]

