Useful Work & Exergy Concepts

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ME 407: Advanced Thermodynamics

`http://zahurul.buet.ac.bd/ME407/`

Overview

Energy: Quantity & Quality

- Energy has both quantity and quality.
- *Quality of energy* is its potential to produce useful work.
- **First Law of Thermodynamics:**
  energy is conserved in all (non-nuclear) processes.
- **Second Law of Thermodynamics:**
  the quality of energy is reduced in all real processes.

⇒ During transformation and transfer, energy is both conserved and degraded.

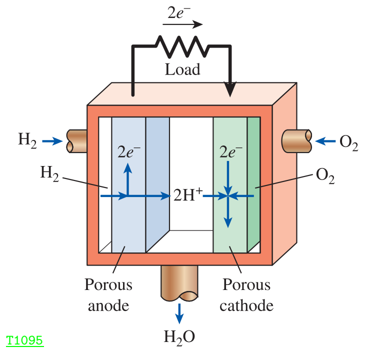
- Exergy (availability) provides a direct relationship between the thermodynamic state of a system and its capability to do useful work.

Datum Condition & Useful Work

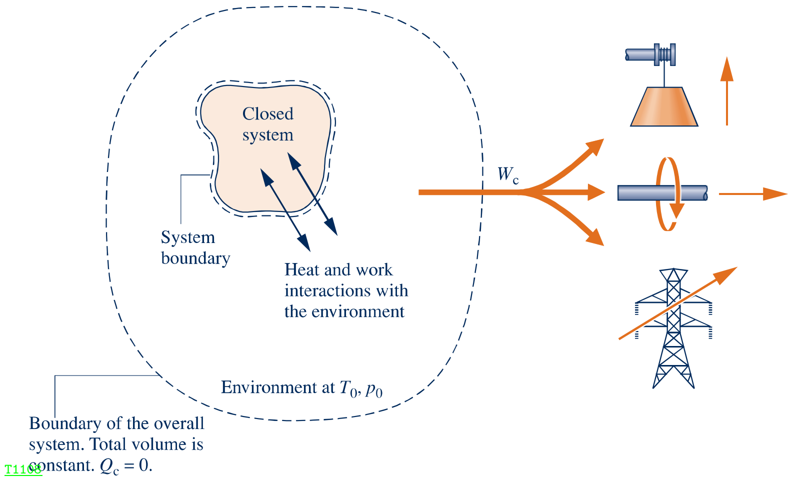
Standard atmosphere:

$P_0 = 100.325$ kPa, $T_0 = 298.15$ K

Species	RH = 60%	RH=100%
N_2	0.7662	0.7564
O_2	0.2055	0.2029
CO_2	0.0003	0.0003
H_2O	0.0188	0.0313
Other	0.0092	0.0091

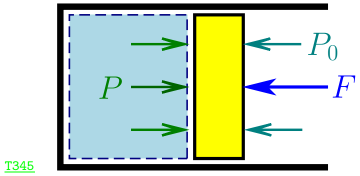


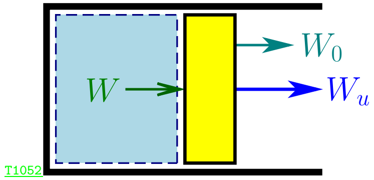
When the pressure, temperature, composition, velocity, or elevation of a system is different from the environment, there is an opportunity to develop work.



Overall system of system and environment used to evaluate exergy.

Useful Work (W_u) & Datum State (T_0, P_0)





Datum state: $P_0 = 101.325$ kPa, $T_0 = 298.15$ K

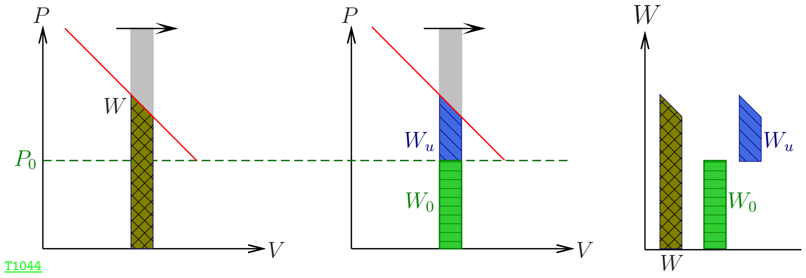
- If $P \approx P_0 \Rightarrow W = \int_1^2 PdV = P_0 \Delta V \neq 0$; But $W_u = 0$.
- $\delta W_u = \vec{F}.d\vec{x} = (P - P_0)Adx = (P - P_0)dV = \delta W - P_0 dV$

$$\Rightarrow \boxed{\delta W_u = \delta W - P_0 dV = \delta W - \delta W_0 \qquad \Rightarrow W_u = W - W_0}$$

- As a closed system expands, some work needs to be done to push the atmospheric air out of the way (and, vice versa) $\rightsquigarrow W_0$.

Useful Work: Expansion & $P > P_0$

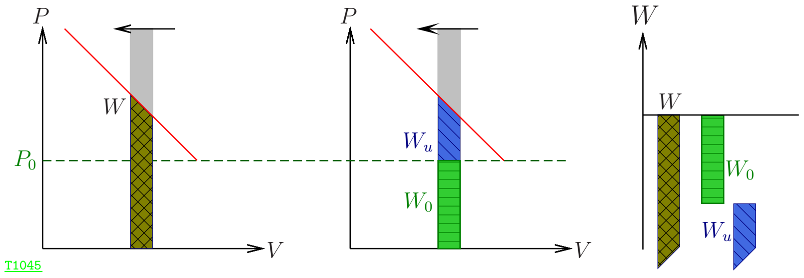
$$\boxed{W_u = W - W_0}$$



Useful work is produced during expansion, if $P > P_0$.

Useful Work: Compression & $P > P_0$

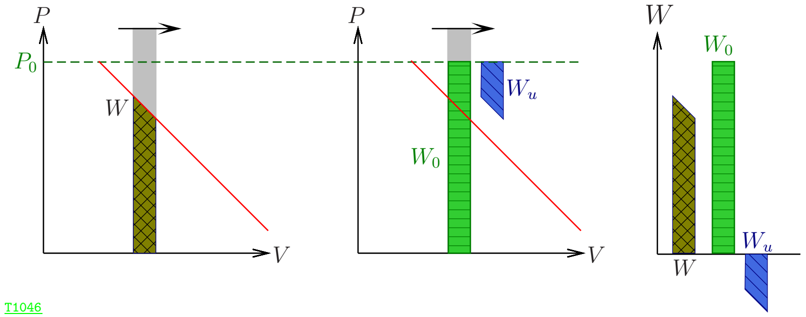
$$\boxed{W_u = W - W_0}$$



Useful work must be supplied during compression, if $P > P_0$.

Useful Work: Expansion & $P < P_0$

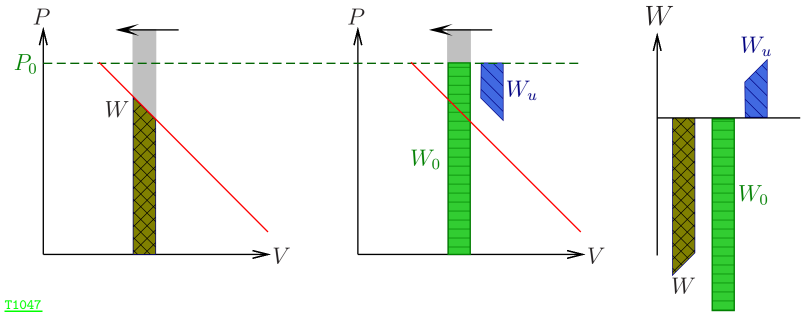
$$\boxed{W_u = W - W_0}$$



Useful work must be supplied during expansion, if $P < P_0$.

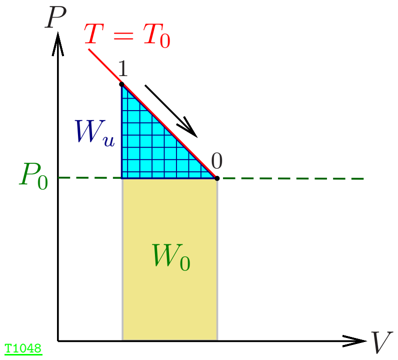
Useful Work: Compression & $P < P_0$

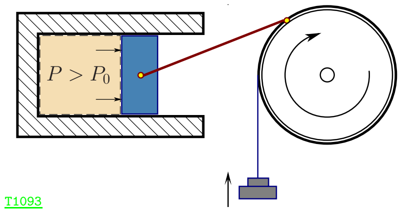
$$\boxed{W_u = W - W_0}$$



Useful work is produced during compression, if $P < P_0$.

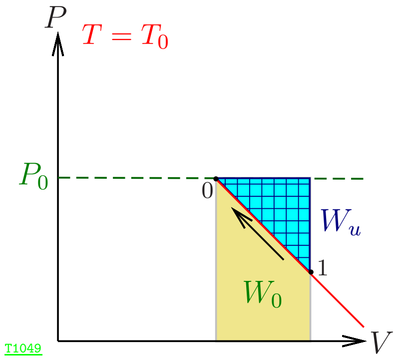
Case I: $P > P_0$, $T = T_0$

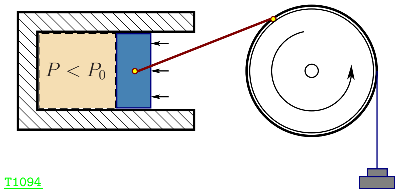




Case I: Isothermal expansion to P_0 to produce useful work.

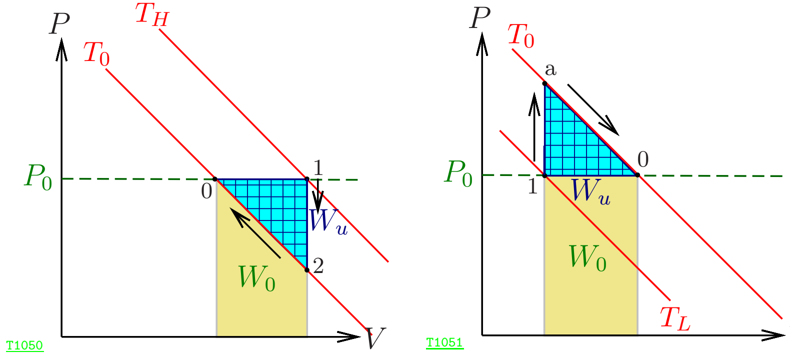
Case II: $P < P_0$, $T = T_0$





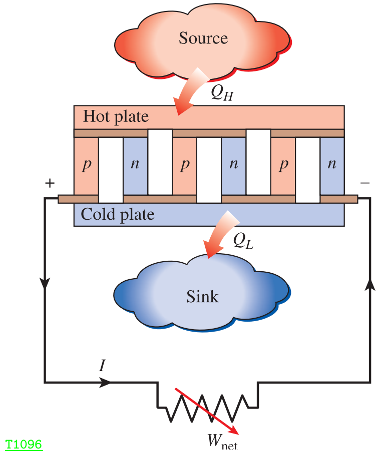
Case II: Isothermal compression to P_0 to produce useful work.

Case III: $P = P_0$, $T = T_H$, Case IV: $P = P_0$, $T = T_L$



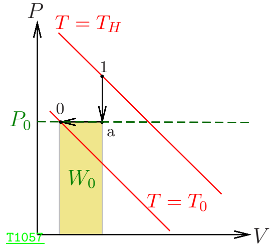
Case III: system could be cooled to T_0 at constant volume, and then compressed isothermally to produce useful work.

Case IV: system could absorb heat to T_0 at constant volume, and then expand isothermally to produce useful work.

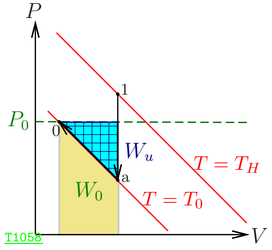


Useful work could be produced by utilizing temperature deviation from the environment.

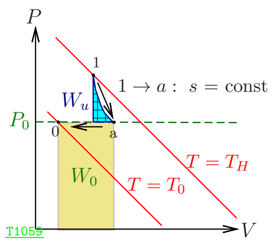
Example: W_u depends on path



$W_u = 0$

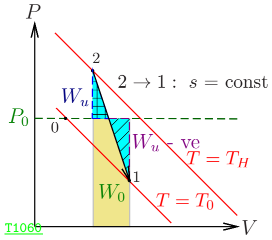


$W_u = +ve$

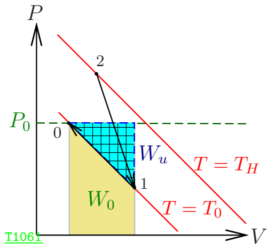


$W_u = +ve$

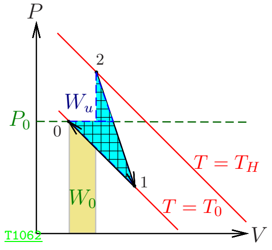
Example: Net W_u



$2 \to 1$

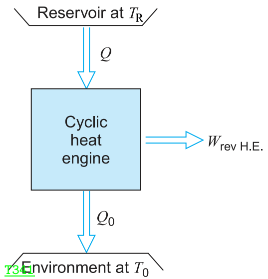


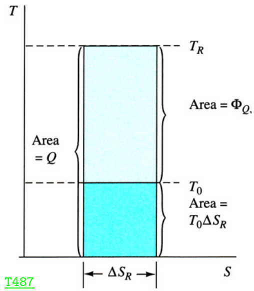
$1 \to 0$



$2 \to 1 \to 0$: net W_u

Exergy of Heat Transfer (Φ_Q)

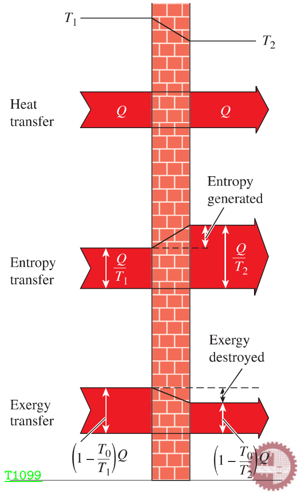


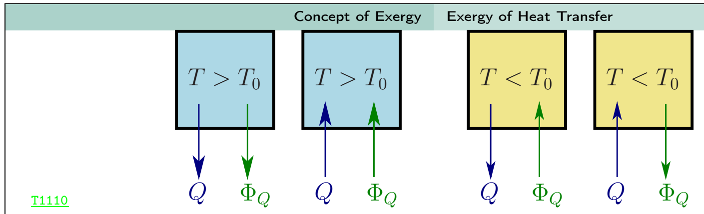


$$\Phi_Q \equiv W_{rev,HE} = Q\left(1-\frac{T_0}{T_R}\right)$$

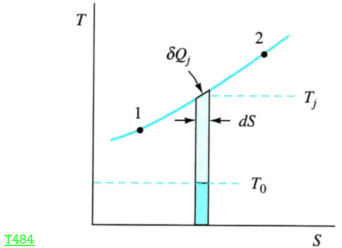
$$\Phi_Q = \textstyle\sum_{j=0}^{n} Q_j\left(1-\frac{T_0}{T_j}\right)$$

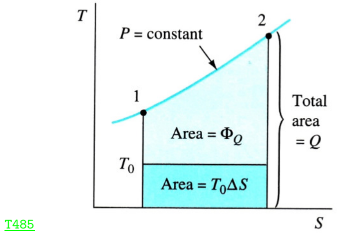
$$I_Q = \Phi_{Q,1} - \Phi_{Q,2} = T_0 Q\left[\frac{1}{T_2}-\frac{1}{T_1}\right] = T_0\sigma_Q$$





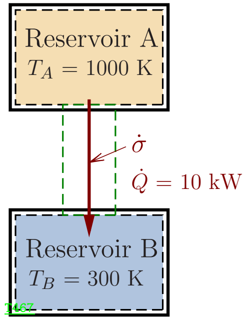
Directions of Q and Φ_Q in relation to T and T_0





$$\Phi_Q = \textstyle\int_1^2 \left(1-\frac{T_0}{T_j}\right)\delta Q_j$$

Example:Heat Transfer ▷ Estimate (a) entropy production (b) entropy change (c) loss in work potential, (d) exergy transfers, (e) irreversibility.



(a) $\dot\sigma_Q = \frac{\dot Q}{T_A T_B}(T_A - T_B) = 0.023$ kW/K

(b) $\frac{dS}{dt} = \frac{dS_A}{dt} + \cancelto{0}{\frac{dS_Q}{dt}} + \frac{dS_B}{dt}$

$\Rightarrow \frac{dS_A}{dt} = -\frac{Q_{A,out}}{T_A} = -10/1000 = -0.01$ kW/K

$\Rightarrow \frac{dS_B}{dt} = \frac{Q_{B,in}}{T_B} = +10/300 = 0.033$ kW/K

$\Rightarrow \frac{dS}{dt} = -0.01 + 0.033 = 0.023$ kW/K

$\Rightarrow \frac{dS}{dt} = \dot\sigma_Q = 0.023$ kW/K

(c) $\dot W_{loss,Q} = T_0\dot\sigma_Q = 300(0.023) = 7.0$ kW

(d) $\Phi_{Q,1000} = 10\left(1-\frac{300}{1000}\right) = 7.0$ kW, $\Phi_{Q,300} = 0$

(e) $I_Q = \Phi_{Q,1000} - \Phi_{Q,300} = 7.0$ kW

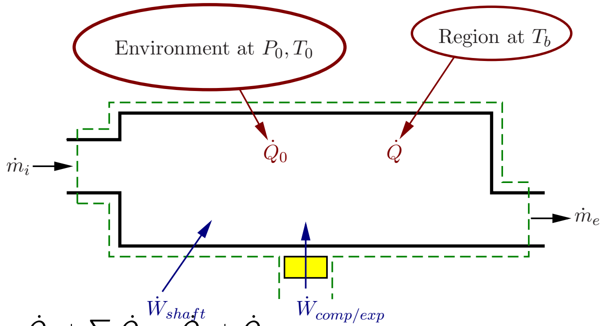
Work & Entropy Production

① $\frac{dE_{cv}}{dt} = \dot Q_{cv} - \dot W_{cv} + \sum_i \dot m_i\left[h + \frac{\mathbb{V}^2}{2} + gz\right]_i - \sum_e \dot m_e\left[h + \frac{\mathbb{V}^2}{2} + gz\right]_e$

② $\frac{dS_{cv}}{dt} = \sum\frac{\dot Q_j}{T_j} + \sum_i(\dot m s)_i - \sum_e(\dot m s)_e + \dot\sigma_{cv}$

- $\dot W_{cv} \equiv$ rate of all possible forms of work transfer.
- $\delta W_u \equiv \delta W_{cv} - \delta W_0 = \delta W_{cv} - P_0 dV \longrightarrow \dot W_{cv} = \dot W_u + P_0\frac{dV_{cv}}{dt}$
- ① - ② $\times T_0$: to make the equations dimensionally consistent.

$$\dot W_u = \left[\dot Q_{cv} - T_0\sum_{j=0}^{n}\frac{\dot Q_j}{T_j}\right] - \frac{d}{dt}\left[E + P_0 V - T_0 S\right]_{cv} - T_0\dot\sigma_{cv}$$
$$+ \sum_i \dot m_i\left[h + \frac{\mathbb{V}^2}{2} + gz - T_0 s\right]_i - \sum_e \dot m_e\left[h + \frac{\mathbb{V}^2}{2} + gz - T_0 s\right]_e$$



T338

- $\dot{Q}_{cv} = \dot{Q}_0 + \sum \dot{Q}_j = \dot{Q}_0 + \dot{Q}$
- $\sum_{i=0}^{n} \frac{\dot{Q}_j}{T_j} = \frac{\dot{Q}_0}{T_0} + \sum_{i=1}^{n} \frac{\dot{Q}_j}{T_j} \simeq \frac{\dot{Q}_0}{T_0} + \frac{\dot{Q}}{T_b}$: $T_b \equiv$ av. boundary temp.

$$\begin{aligned}\dot{W}_u &= \dot{Q}\left[1 - \frac{T_0}{T_b}\right] - \frac{d}{dt}\left[E + P_0 V - T_0 S\right]_{cv} - T_0 \dot{\sigma}_{cv} \\ &+ \sum_i \dot{m}_i \left[h + \frac{\mathbb{V}^2}{2} + gz - T_0 s\right]_i - \sum_e \dot{m}_e \left[h + \frac{\mathbb{V}^2}{2} + gz - T_0 s\right]_e\end{aligned}$$

Exergy of Closed System ($\dot{m}_i = \dot{m}_e = 0$)

For CM system: $\dot{W}_{u,rev} = \dot{Q}\left[1 - \frac{T_0}{T_b}\right] - \frac{d}{dt}\left[E + P_0 V - T_0 S\right]_{cm}$

$\Rightarrow \delta W_{u,rev} = \delta Q\left[1 - \frac{T_0}{T_b}\right] - d\left[E + P_0 V - T_0 S\right]_{cm}$

If system exchanges heat solely with environment at T_0:

- $\delta W_{u,rev} = -d\left[E + P_0 V - T_0 S\right]_{cm} = -d\left[U + P_0 V - T_0 S\right]_{cm} = -dA$
- $\boxed{A \equiv U + P_0 V - T_0 S}$ $\Phi \equiv$ **availability of closed system**.
- For a CM system, from a given state to dead state (P_0, T_0):

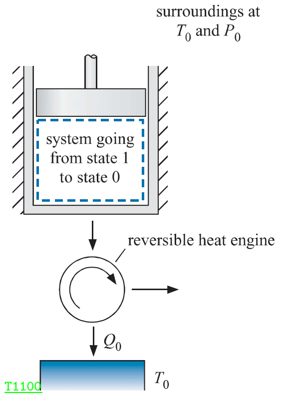
$\Rightarrow$ $W_{u,rev} = -(A_0 - A) = A - A_0$

$$\boxed{\Phi \equiv W_{u,rev,out} = A - A_0}$$

$$\boxed{① \rightarrow ② : \; W_{u,rev,1\rightarrow 2} = -(A_2 - A_1) = -(\Delta U + P_0 \Delta V - T_0 \Delta S) = -\Delta\Phi}$$

$$\boxed{\phi \equiv \frac{\Phi}{m} \equiv (u - u_0) + P_0(v - v_0) - T_0(s - s_0) = a - a_o}$$

$$\boxed{w_{u,rev,1\rightarrow 2} = -\Delta\phi = (\phi_1 - \phi_2)}$$



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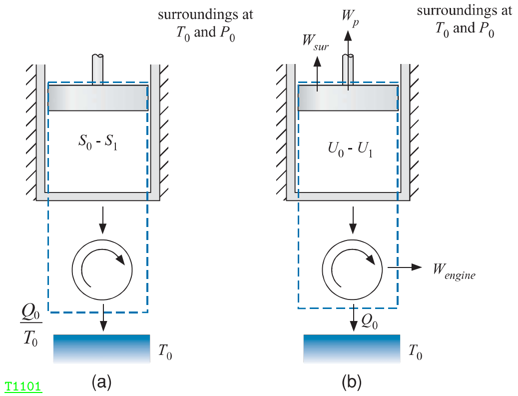
Extracting maximum W_u



T1101

(a) (b)

(a) Energy balance, (b) Entropy balance

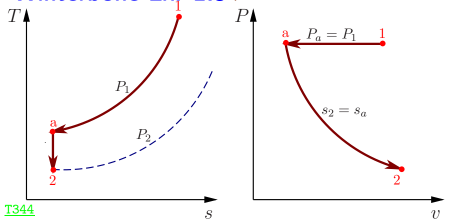
- **(Thermo-mechanical/physical) Exergy of a CM system** in a given state is defined as the maximum work output that might be obtained from a system-environment combination as the system proceeds from a given equilibrium state to the **restricted dead state** by a process where any heat-transfer occurs only with the environment.
- At **restricted dead state** the control mass is in thermal and mechanical equilibrium with the environment, but not necessarily in chemical equilibrium with it.
- The difference between the composition of the control mass at restricted dead state and that of the environment can be exploited (by permitting to mix with the environment or enter into chemical reaction with the environmental components) to obtain additional work. The maximum work obtainable in this way is the **chemical exergy**.
- Total exergy is the sum of physical exergy and chemical exergy.

Example: Exergy of Air

- $\phi = (u - u_0) + P_0(v - v_0) - T_0(s - s_0)$
- $u - u_0 = c_v(T - T_0)$
- $v = \frac{RT}{P}$
- $s - s_0 = c_V \ln\left(\frac{T}{T_0}\right) + R\ln\left(\frac{v}{v_0}\right) = c_P \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{P_2}{P_1}\right)$
- Environment: $T_0 = 298.15$ K, $P_0 = 101.325$ kPa.
  - ▷ Air at 298.15 K & 101.325 kPa: $\phi = 0$ kJ/kg
  - ▷ Air at 298.15 K & 50 kPa: $\phi = 27.4$ kJ/kg
  - ▷ Air at 298.15 K & 200 kPa: $\phi = 16.0$ kJ/kg
  - ▷ Air at 200 K & 101.325 kPa: $\phi = 20.8$ kJ/kg
  - ▷ Air at 400 K & 101.325 kPa: $\phi = 14.4$ kJ/kg

When the pressure, temperature, composition, velocity, or elevation of a system is different from the environment, there is an opportunity to develop work.

Winterbone Ex. 2.3 ▷



- $P_1 = 200$ kPa, $T_1 = 550$ K
- $P_2 = P_0 = 100$ kPa, $T_2 = T_0 = 300$ K

$\Rightarrow T_a = T_2 \left[\frac{P_a}{P_2}\right]^{(k-1)/k} = 366$ K

- $v = RT/P \Rightarrow v_1 = 0.7896,\ v_a = 0.5248$ & $v_2 = 0.861$ m^3/kg
- $w_{sys}|_{1\to a} = \int_1^a Pdv = P_1(v_a - v_1) = -52.96$ kJ/kg
- $w_{surr}|_{1\to a} = \int_1^a P_0 dv = P_0(v_a - v_1) = -26.46$ kJ/kg
- $w_u|_{1\to a} = w_{sys}|_{1\to a} - w_{surr}|_{1\to a} = -26.46$ kJ/kg
- $w_{sys}|_{a\to 2} = \int_a^2 Pdv = \frac{R}{1-k}(T_2 - T_a) = +47.14$ kJ/kg
- $w_{surr}|_{a\to 2} = \int_a^2 P_0 dv = P_0(v_2 - v_a) = +33.63$ kJ/kg
- $w_u|_{a\to 2} = w_{sys}|_{a\to 2} - w_{surr}|_{a\to 2} = +13.51$ kJ/kg

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- $w_u|_{1\to 2} = w_u|_{1\to a} + w_u|_{a\to 2} = -12.95$ kJ/kg
- $1 \to a$: temperature is decreased from 550 K to 365.7 K and heat transfer to surroundings occurs. It can be used to produce useful work by reversible heat engine.
  - $\delta w_R = -\eta_R \delta q = -(1 - T_0/T)\delta q$
  - $w_R|_{1\to a} = -\int_1^a (1 - T_0/T)\delta q = -C_p \int_1^a (1 - T_0/T)dT = +62.18$ kJ/kg
  - $w_R|_{a\to 2} = 0$ kJ/kg as no heat transfer occurs.
  - $w_R|_{1\to 2} = w_R|_{1\to a} + w_R|_{a\to 2} = +62.18$ kJ/kg
- $w_{u,max}|_{1\to 2} = w_u|_{1\to 2} + w_R|_{1\to 2} = +49.23$ kJ/kg◀
- **Exergy approach:**
  $w_{u,max}|_{1\to 2} = -\Delta\phi = -[(u_2 - u_1) + P_0(v_2 - v_1) - T_0(s_2 - s_1)]$
  - $u_2 - u_1 = c_v(T_2 - T_1) = -179.5$ kJ/kg
  - $P_0(v_2 - v_1) = 7.14$ kJ/kg
  - $T_0(s_2 - s_1) = T_0\left\{c_P \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{P_2}{P_1}\right)\right\} = -123.7$ kJ/kg
  - $w_{u,max}|_{1\to 2} = +49.28$ kJ/kg◀

Exergy Balance of a CM System

- $\delta W_u = \delta Q\left[1 - \frac{T_0}{T_b}\right] - d\left[E + P_0 V - T_0 S\right]_{cv} - T_0 \delta\sigma_{cm}$

$$\begin{aligned} W_u &= \int_1^2 \left[1 - \frac{T_0}{T_b}\right]\delta Q - \left[\Delta U + P_0 \Delta V - T_0 \Delta S\right] - T_0 \sigma_{cm} \\ &= \Phi_Q - \Delta\Phi - I_{cm} \end{aligned}$$

$$\boxed{\Delta\Phi = \Phi_Q - W_u - I_{cm}}$$

$\Delta\Phi$ = exergy change of a CM system

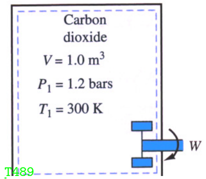
Φ_Q = exergy transfer with heat transfer into system

W_u = exergy transfer with useful work out of system

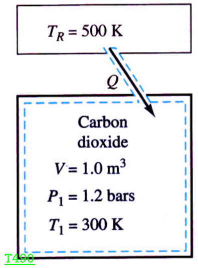
$I_{cm} \equiv T_0\sigma_{cm}$ = exergy destruction within CM

- $W_{u,rev} = W_u\,|_{I_{cm}=0} = \Phi_Q - \Delta\Phi \;:\Rightarrow \boxed{I_{cm} = W_{u,rev} - W_u}$
- Heat transfer region $:\Rightarrow \boxed{I_Q = \Phi_{Q,in} - \Phi_{Q,out}}\ \because \Delta\Phi = W_u = 0$
- For isolated system $:\Rightarrow \boxed{\Delta\Phi = -I_{cm} \longmapsto \Delta\Phi \leqslant 0}$

Wark(1999), Ex. 9.5: ▷ Temperature is increased to 400 K using (a) using a paddle wheel (b) using heat transfer from a source at 500 K. Irreversibility = ?



T489



T490

- $R = R_u/M = 188.9$ J/kg-K, $m = \frac{P_1V_1}{RT_1} = 2.12$ kg
- $\Phi \equiv (u - u_0) + P_0(v - v_0) - T_0(s - s_0)$
- $\Delta\Phi = \Phi_Q - W_u - I_{cm}$
- **(a):** $W_u = W_{12} = m(u_2 - u_1) =$ -149.1 kJ
- $\Phi_Q = 0.0$: no heat transfer
- $\Delta\Phi = 20.4$ kJ/kg $\Rrightarrow I_{cm} = 128.7$ kJ ◀
- **(b):** $T_b = T_R = 500$ K, $Q_{12} = m(u_2 - u_1) = 149.1$ kJ
- $W_u = W_{12} = 0.0$
- $\Phi_Q = Q_{12}(1.0 - T_0/T_b) = 59.6$ kJ
- $\Delta\Phi = 20.4$ kJ/kg $\Rrightarrow I_{cm} = 39.2$ kJ ◀

Exergy of CV System

$$\dot{W}_u = \sum_{j=1}^{n} \dot{Q}_j \left[1 - \frac{T_0}{T_j}\right] - \frac{d}{dt}\left[E + P_0V - T_0S\right]_{cv} - T_0\dot{\sigma}_{cv} + \sum_i \dot{m}_i \left[h + \frac{\mathbb{V}^2}{2} + gz - T_0s\right]_i - \sum_e \dot{m}_e \left[h + \frac{\mathbb{V}^2}{2} + gz - T_0s\right]_e$$

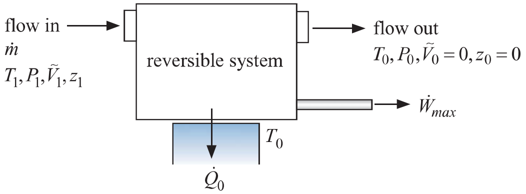
- For SSSF process: $\frac{d}{dt}\left[E + P_0V - T_0S\right]_{cv} = 0$ & $P_0\Delta V = 0$.

$\Rightarrow$ $\dot{W}_u = \dot{W}_{sf}$ & $\dot{W}_{sf,rev} = \dot{W}_{sf} + T_0\dot{\sigma}_{cv}$

- **Flow exergy (ψ)** of a fluid in steady flow is defined as the maximum work output that can be obtained as the fluid is changed reversibly from the given state to a dead state in a process where any heat transfer occurs solely with the environment.

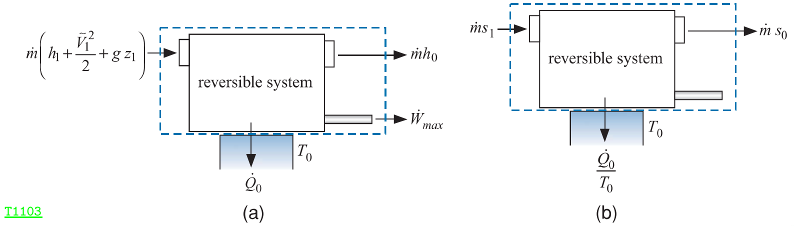
$$\Rightarrow \boxed{\psi \equiv w_{u,rev} \equiv (h - h_0) - T_0(s - s_0) + \frac{\mathbb{V}^2}{2} + gz}$$

$$\Rightarrow \boxed{\dot{W}_{sf,rev} = \sum_{j=1}^{n} \dot{Q}_j \left[1 - \frac{T_0}{T_j}\right] + \sum_i \dot{m}_i\psi_i - \sum_e \dot{m}_e\psi_e}$$



T1102

Extracting maximum useful work



T1103

(a) Energy balance, (b) Entropy balance

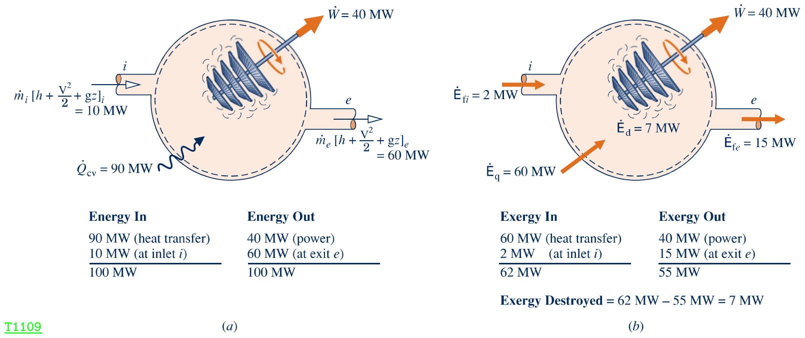
Exergy Balance of a CV System: SSSF Process

$$\dot{W}_{u,act} = \sum_{j=1}^{n} \dot{Q}_j \left[1 - \frac{T_0}{T_j}\right] - \frac{d}{dt}\left[E + P_0V - T_0S\right]_{cv} - T_0\dot{\sigma}_{cv} + \sum_i \dot{m}_i \left[h + \frac{\mathbb{V}^2}{2} + gz - T_0s\right]_i - \sum_e \dot{m}_e \left[h + \frac{\mathbb{V}^2}{2} + gz - T_0s\right]_e$$

$\Rightarrow$ $\dot{W}_{u,act} = \Phi_Q - 0 - \dot{I}_{cv} + \sum_i \dot{m}_i\psi_i - \sum_e \dot{m}_e\psi_e$

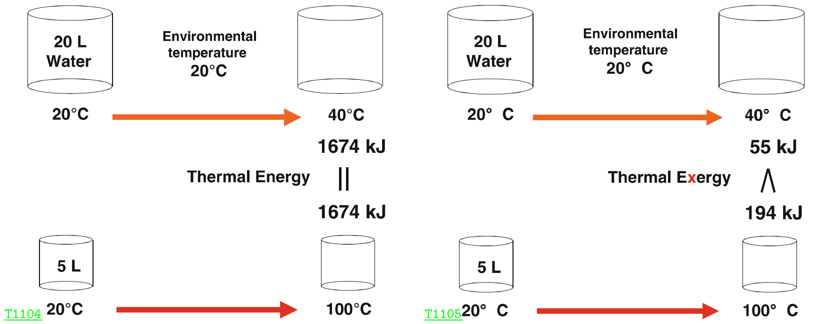
$$\Rightarrow \boxed{\Delta(m\psi) = \sum_e \dot{m}_e\psi_e - \sum_i \dot{m}_i\psi_i = \dot{\Phi}_Q - \dot{W}_{u,act} - \dot{I}_{cv}}$$

$\Delta(\dot{m}\psi)$: Net exergy outflux rate with mass flow.
$\dot{\Phi}_Q$: Net exergy transfer rate with heat transfer into CV.
$\dot{W}_{u,act}$: Net exergy transfer rate with work out of CV.
$\dot{I}_{cv}$: Net exergy destruction rate within CV.



T1109

Comparing energy and exergy for a control volume at steady state. (a) Energy analysis. (b) Exergy analysis.



20 L Water

Environmental temperature 20°C

20°C → 40°C

1674 kJ

Thermal Energy ||

1674 kJ

5 L

T1104 20°C → 100°C

20 L Water

Environmental temperature 20° C

20° C → 40° C

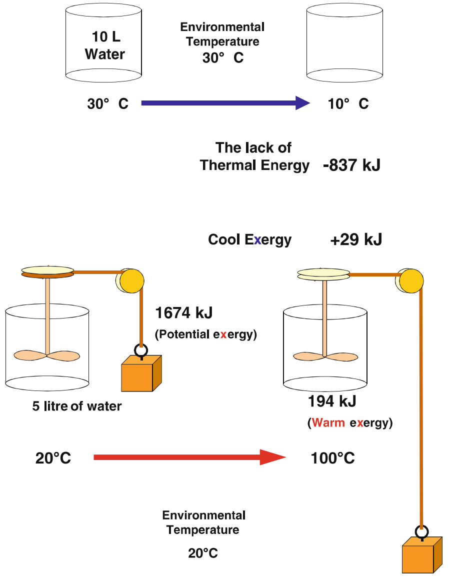
55 kJ

Thermal Exergy ⋀

194 kJ

5 L

T1105 20° C → 100° C



10 L Water

Environmental Temperature 30° C

30° C → 10° C

The lack of Thermal Energy -837 kJ

Cool Exergy +29 kJ

T1106

1674 kJ (Potential exergy)

5 litre of water

194 kJ (Warm exergy)

20°C → 100°C

Environmental Temperature 20°C

T1107

Characteristics of Exergy

- Exergy is the maximum work that can be extracted from a combined system and the environment.
- At datum state, both system and the environment posses energy, but the exergy is zero.
- Exergy is never negative.
- Exergy is a measure of deviation from the state of the system from that of the environment. The greater the deviation, the higher the value of exergy.
- Exergy is destroyed by irreversibilities.