Heat Transfer with Internal Heat Generation

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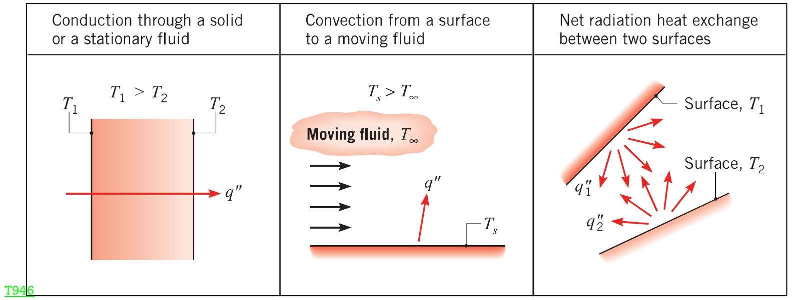
ME 307: Heat Transfer Equipment Design
`http://zahurul.buet.ac.bd/ME307/`

Modes of Heat Transfer

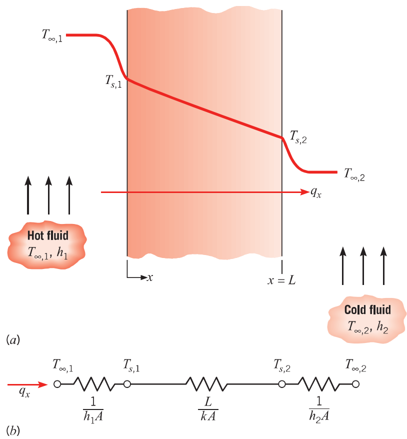


T946

System with Internal Heat Generation

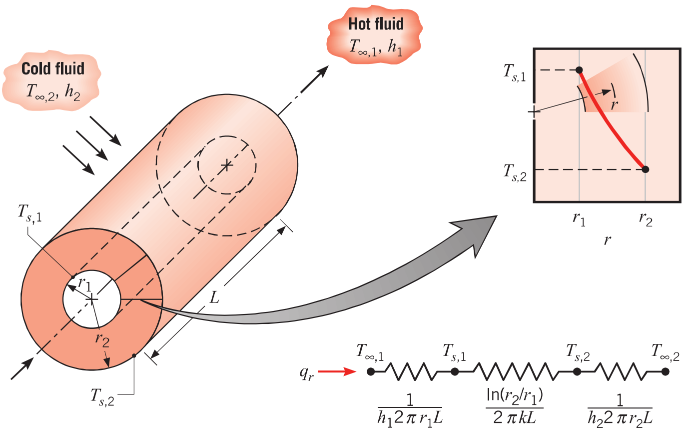
- Nuclear reactors
- Electric heater & conductors
- Exothermic chemical reactions

Heat Conduction & Convection



T740

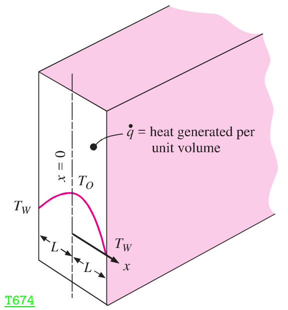
Heat transfer through a plane wall. (a) Temperature distribution. (b) Equivalent thermal circuit.



T741

Hollow cylinder with convective surface conditions.

Plane Wall with Uniform Heat Generation



- DE: $\frac{d^2T}{dx^2} + \frac{\dot{q}_g}{k} = 0$

$\Rightarrow$ $T(x) = -\frac{\dot{q}_g}{2k}x^2 + C_1 x + C_2$

- BC: $T = T_w$ at $x \pm L$

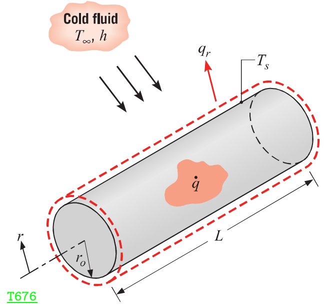
$\Rightarrow$ $$\boxed{T(x) = T_w + \frac{\dot{q}_g}{2k}L^2\left[1 - \left(\frac{x}{L}\right)^2\right]}$$

- if $T(x = 0) = T_0$:

$\Rightarrow$ $$\boxed{T_0 = T_w + \frac{\dot{q}_g}{2k}L^2}$$

$\Rightarrow$ $$\boxed{\frac{T - T_w}{T_0 - T_w} = 1 - \left(\frac{x}{L}\right)^2}$$

Radial System with Internal Heat Generation



- DE: $\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{\dot{q}_g}{k} = 0$

$\Rightarrow$ $T(r) = -\frac{\dot{q}_g}{4k}r^2 + C_1 \ln r + C_2$

- BCs:
  - $T(r_0) = T_s$
  - $\frac{dT}{dr}|_{r=0} = 0$

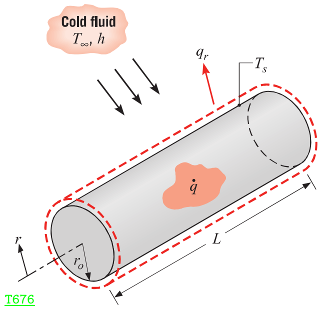
$\Rightarrow$ $$\boxed{T(r) = T_s + \frac{\dot{q}_g}{4k}r_0^2\left[1 - \left(\frac{r}{r_0}\right)^2\right]}$$

- if $T(r = 0) = T_0$:

$\Rightarrow$ $$\boxed{T_0 = T_s + \frac{\dot{q}_g}{4k}r_0^2}$$

$\Rightarrow$ $$\boxed{\frac{T(r) - T_s}{T_0 - T_s} = 1 - \left(\frac{r}{r_0}\right)^2}$$

contd. . . .



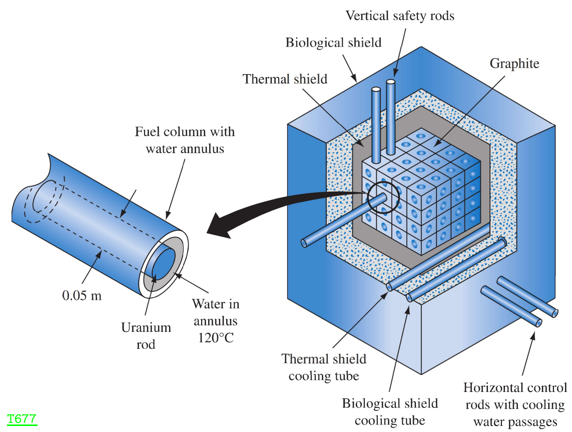
- Applying an overall energy balance:

$$\dot{q}_g(\pi r_0^2 L) = h(2\pi r_0 L)(T_s - T_\infty)$$

$\Rightarrow$ $$\boxed{T_s = T_\infty + \frac{\dot{q}_g r_0}{2h}}$$

$$\boxed{T_{max} = T_0 = T_s + \frac{\dot{q}_g}{4k}r_0^2 = T_\infty + \frac{\dot{q}_g r_0}{4h}\left(2 + \frac{hr_o}{k}\right)}$$

Example: ▷ Heat is generated at 7.5×10^7 W/m^3 in nuclear reactor uranium rods of 0.05 m diameter. These rods are jacketed by an annulus in which water at 120°Cis circulated. If h_{av} = 55000 W/m^2 K, and for uranium, k = 29.5 W/m K, determine the center temperature of the uranium fuel rods.

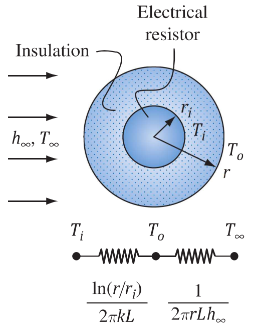


- $T_s = 137.05^0$C ◀
- $T_0 = 534.29^0$C ◀

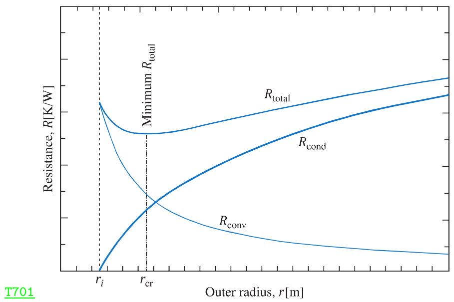
Example: ▷ A current of 200 A is passed through a stainless-steel wire [k =19 W/m°C] 3 mm in diameter. The resistivity of the steel may be taken as 70 μΩcm, and the length of the wire is 1 m. The wire is submerged in a liquid at 110°Cand experiences a convection heat-transfer coefficient of 4 kW/m^2°C. Calculate the center temperature of the wire.

- $\dot{q}_g = 560.39$ MW/m^3 ◀
- $T_o = 231.66$°C ◀

Critical Thickness of Insulation



T700

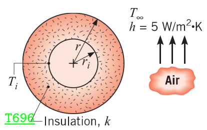


T701

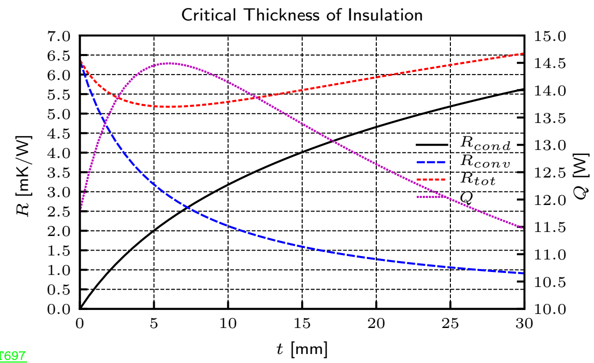
- $R_{total} = R_{cond} + R_{conv} = \frac{\ln(r/r_i)}{2\pi kL} + \frac{1}{2\pi rLh_\infty}$
- For maximum heat loss, $\frac{dR_{total}}{dr} = \frac{1}{2\pi krL} - \frac{1}{2\pi r^2 Lh_\infty} = 0 \rightarrow r = \frac{k}{h_\infty}$

Critical thickness of insulation, $r_{cr} = \frac{k}{h_\infty}$

Example: ▷ Critical thickness of insulation, $k = 0.055$ W/mK, $r_i =$ 5mm, T_i = 100°C, $T_\infty =$ 25°C.



T696



T697

- $r_{cr} = k/h = 11$ mm, $t = r_{cr} - r_i = 6$ mm◀
- $Q = 14.5$ W◀