Heat Transfer from Extended Surfaces

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ME 307: Heat Transfer Equipment Design

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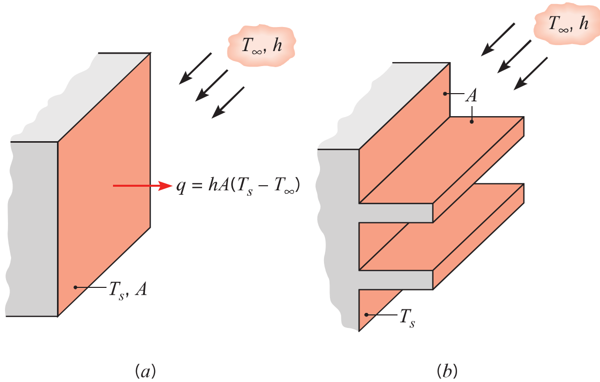
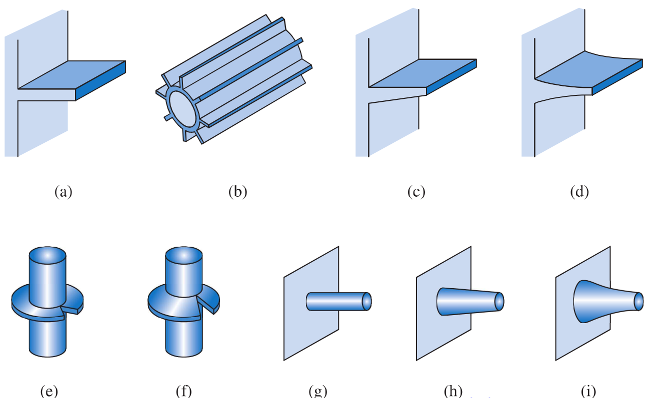


T678

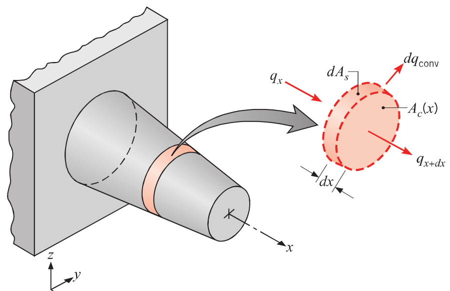
Use of fins to enhance the heat transfer from a plane wall. (a) Bare surface, (b) Finned surface.



T689

Schematic diagrams of different types of fins: (a) longitudinal fin of rectangular profile; (b) cylindrical tube with fins of rectangular profile; (c) longitudinal fin of trapezoidal profile; (d) longitudinal fin of parabolic profile; (e) cylindrical tube with radial fin of rectangular profile; (f) cylindrical tube with radial fin of truncated conical profile; (g) cylindrical pin fin; (h) truncated conical spine; (i) parabolic spine.

General Heat Transfer Equation for Extended Surfaces

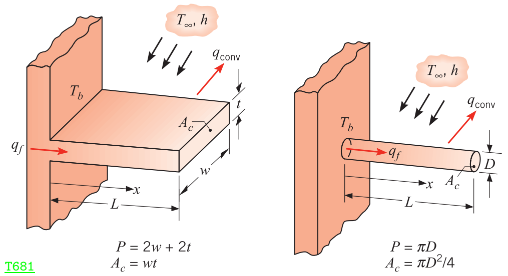


T680

- Energy balance: $q_x = q_{x+dx} + dq_{conv}$
- Fourier's law of conduction: $q_x = -kA_c\frac{dT}{dx}$
- Newton's law of cooling: $dq_{conv} = h_c dA_s(T - T_\infty)$

$$\Rightarrow \boxed{\frac{d^2T}{dx^2} + \left(\frac{1}{A_c}\frac{dA_c}{dx}\right)\frac{dT}{dx} - \left(\frac{1}{A_c}\frac{h_c}{k}\frac{dA_s}{dx}\right)(T - T_\infty) = 0}$$

Fins of Uniform Cross-Sectional Area



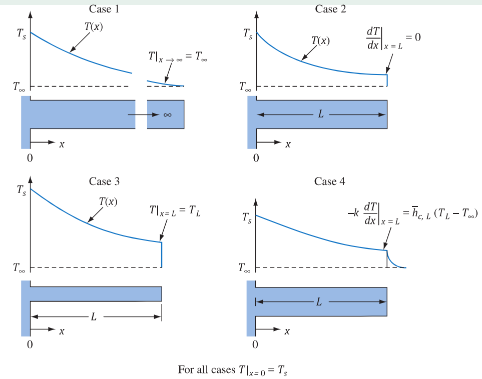
T681

- A_c = Constant.
- $A_s = Px$
- $\theta(x) \equiv T - T_\infty$
- $m^2 \equiv \frac{h_c P}{kA_c}$
- $\theta|_{x=0} \equiv \theta_s = T_s - T_\infty$

- $\frac{d^2 T}{dx^2} - \frac{h_c P}{kA_c}(T - T_\infty) = 0$

$\Rightarrow$ $$\boxed{\frac{d^2\theta}{dx^2} - m^2\theta = 0; \quad \Rightarrow \quad \theta(x) = C_1 e^{mx} + C_2 e^{-mx}}$$

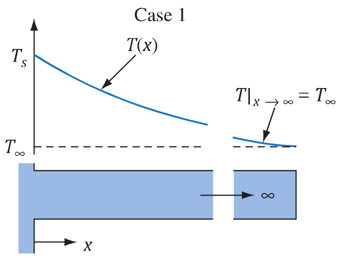
Fin: Different Boundary Conditions



T687

1. Long fin, $\theta = 0$, $at\ x \to \infty$
2. End of the fin is insulated, $\frac{d\theta}{dx} = 0$, $at\ x = L$
3. End temperature is fixed, $\theta = \theta_L$, $at\ x = L$
4. Fin end loses heat by convection, $-k\frac{d\theta}{dx}|_{x=L} = h_c\theta_L$

1. Long Fin, $\theta = 0$, $at\ x \to \infty$



T690

- $\frac{d^2\theta}{dx^2} - m^2\theta = 0$
  1. $\theta|_{x=0} \equiv \theta_s = T_s - T_\infty$
  2. $\theta|_{x\to\infty} = 0$

$\Rightarrow$ $$\boxed{\theta(x) = C_1 e^{mx} + C_2 e^{-mx}}$$

- BC ①: $C_1 + C_2 = \theta_s$
- BC ②: $C_1 = 0$, $\because\ \theta(x \to \infty) = 0$

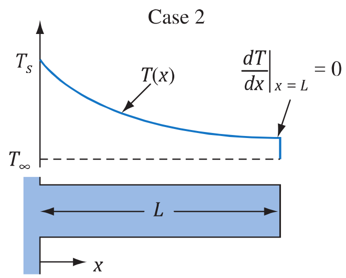
$\Rightarrow$ $$\boxed{\frac{\theta(x)}{\theta_s} = \frac{T(x) - T_\infty}{T_s - T_\infty} = e^{-mx}}$$

- $Q = -kA_c\frac{d\theta(x)}{dx}|_{x=0}$

$\Rightarrow$ $$\boxed{Q = kA_c\theta_s m = \theta_s\sqrt{h_c PkA_c}}$$

- $M \equiv \theta_s\sqrt{h_c PkA_c}$

2. Insulated tip, $\frac{d\theta}{dx} = 0$, $at\ x = L$



T691

- $\frac{d^2\theta}{dx^2} - m^2\theta = 0$
  1. $\theta|_{x=0} \equiv \theta_s = T_s - T_\infty$
  2. $\frac{d\theta}{dx}|_{x=L} = 0$

$\Rightarrow$ $$\boxed{\theta(x) = C_1 e^{mx} + C_2 e^{-mx}}$$

- BC ①: $C_1 + C_2 = \theta_s$
- BC ②: $mC_1 e^{mL} - mC_2 e^{-mL} = 0$

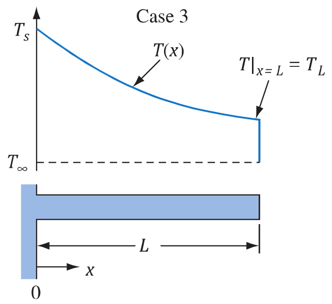
$\Rightarrow$ $$\boxed{\frac{\theta(x)}{\theta_s} = \frac{T(x) - T_\infty}{T_s - T_\infty} = \frac{\cosh m(L - x)}{\cosh mL}}$$

- $Q = -kA_c\frac{d\theta(x)}{dx}|_{x=0}$

$\Rightarrow$ $$\boxed{Q = M \tanh mL}$$

- $M \equiv \theta_s\sqrt{h_c PkA_c}$

3. End temperature is fixed, $\theta = \theta_L$, *at* $x = L$



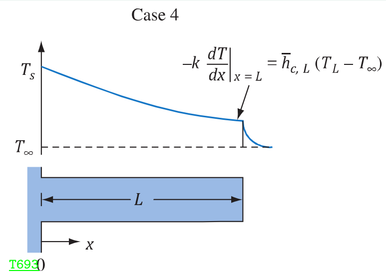
T692

- $\frac{d^2\theta}{dx^2} - m^2\theta = 0$
  1. $\theta|_{x=0} \equiv \theta_s = T_s - T_\infty$
  2. $\theta|_{x=L} = \theta_L$
- $\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$
- BC ①: $C_1 + C_2 = \theta_s$
- BC ②: $C_1 e^{mL} + C_2 e^{-mL} = \theta_L$

$$\Rightarrow \boxed{\frac{\theta(x)}{\theta_s} = \frac{T(x) - T_\infty}{T_s - T_\infty} = \frac{(\theta_L/\theta_s)\sinh mx + \sinh m(L-x)}{\sinh mL}}$$

$$\Rightarrow \boxed{Q = M \frac{\cosh mL - (\theta_L/\theta_s)}{\cosh mL + (h_c/mk)\sinh mL}}$$

4. Fin end loses heat by convection



T693

- $\frac{d^2\theta}{dx^2} - m^2\theta = 0$
  1. $\theta|_{x=0} \equiv \theta_s = T_s - T_\infty$
  2. $\frac{d\theta}{dx}|_{x=L} = 0$
- $\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$
- BC ①: $C_1 + C_2 = \theta_s$
- BC ②: $-km(C_1 e^{mL} - C_2 e^{-mL}) = h_c\theta_L$

$$\Rightarrow \boxed{\frac{\theta(x)}{\theta_s} = \frac{T(x) - T_\infty}{T_s - T_\infty} = \frac{\cosh m(L-x) + (h_c/mk)\sinh m(L-x)}{\cosh mL + (h_c/mk)\sinh mL}}$$

$$\Rightarrow \boxed{Q = M \frac{\sinh mL + (h_c/mk)\cosh mL}{\cosh mL + (h_c/mk)\sinh mL}}$$

Equations for temperature distribution and rate of heat transfer for fins of uniform cross section

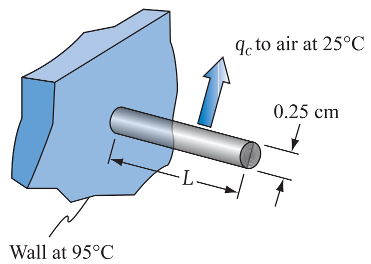
Case	Tip Condition ($x = L$)	Temperature Distribution, θ/θ_s	Fin Heat Transfer Rate, q_{fin}
1	Infinite fin ($L \to \infty$): $\theta(L) = 0$	e^{-mx}	M
2	Adiabatic: $\left.\frac{d\theta}{dx}\right\|_{x=L} = 0$	$\frac{\cosh m(L-x)}{\cosh mL}$	$M \tanh mL$
3	Fixed temperature: $\theta(L) = \theta_L$	$\frac{(\theta_L/\theta_s)\sinh mx + \sinh m(L-x)}{\sinh mL}$	$M\frac{\cosh mL - (\theta_L/\theta_s)}{\sinh mL}$
4	Convection heat transfer: $\bar{h}_c\theta(L) = -k\left.\frac{d\theta}{dx}\right\|_{x=L}$	$\frac{\cosh m(L-x) + (\bar{h}_c/mk)\sinh m(L-x)}{\cosh mL + (\bar{h}_c/mk)\sinh mL}$	$M\frac{\sinh mL + (\bar{h}_c/mk)\cosh mL}{\cosh mL + (\bar{h}_c/mk)\sinh mL}$

$\theta \equiv T - T_\infty$
$\theta_s \equiv \theta(0) = T_s - T_\infty$
$m^2 \equiv \frac{\bar{h}_c P}{kA}$
$M \equiv \sqrt{\bar{h}_c PkA}\ \theta_s$

T688

Example: ▷ Consider a copper ($k = 385$ W/mK) pin with $h_c = 10$ W/m^2 K. Calculate the heat loss, assuming that (a) the fin is 'infinitely long', (b) the fin is 2.5 cm long and the coefficient at the end is the same as around the circumference, (c) how long would the fin have to be for the infinitely long solution to be correct within 5%?

[(a) 0.863 W, (b) 0.140 W, (c) L > 0.284 m.]



T698

Fin Efficiency, η_f

- Fin efficiency,

$$\eta_f \equiv \frac{Q_{fin}}{Q_{ideal}} = \frac{Q_{fin}}{A_s h_c \theta_s}$$

Q_{fin} = actual heat transfer though fin

Q_{ideal} = ideal heat transfer through fin, if entire fin surface were at the base temperature, T_s

Case 2: Insulated tip, $\frac{d\theta}{dx} = 0$, at $x = L$:

$$\eta_f = \frac{M \tanh mL}{A_s h_c \theta_s} = \frac{\tanh mL}{mL}$$

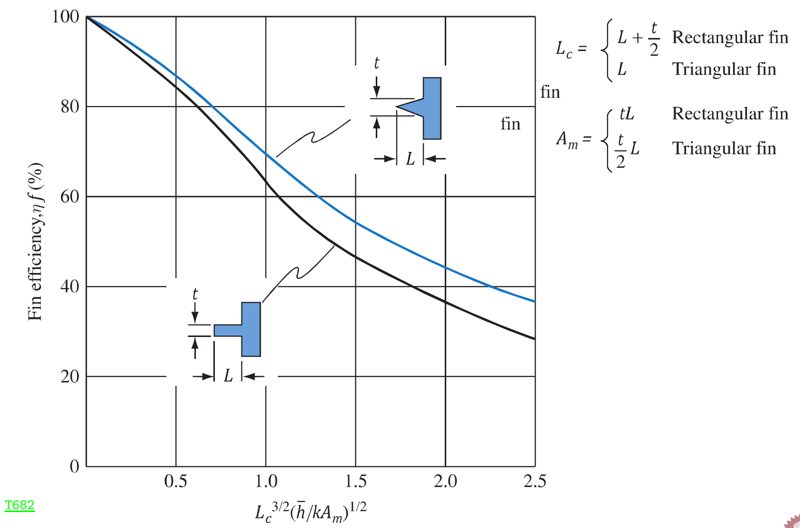
$$\eta_{f,case2} = \begin{cases} 1 & \text{for } L \to 0 \\ 0 & \text{for } L \to \infty \end{cases}$$

- Case 3 & 4: Active tip: Use insulated tip (case 2) formulation with:

$$L_c = \begin{cases} L + t/2 & \text{for rectangular fin} \\ L + D/4 & \text{for circular fin} \end{cases}$$

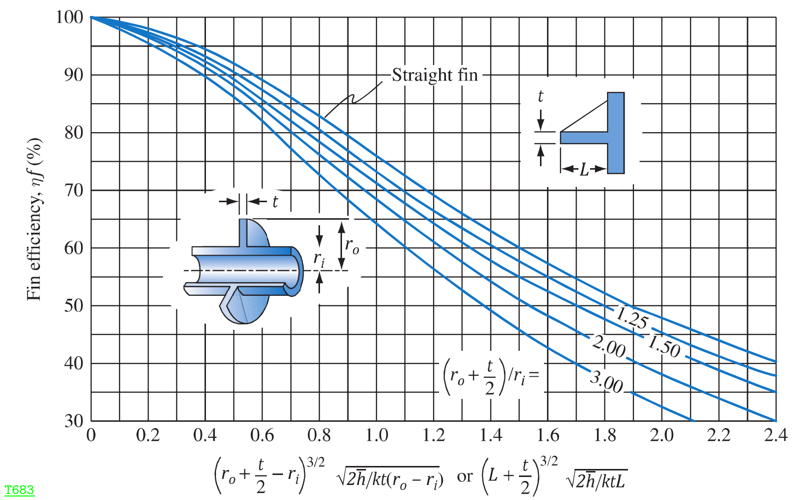
$$\eta_f = \frac{M \tanh mL_c}{A_s h_c \theta_s} = \frac{\tanh mL_c}{mL_c}$$

- For rectangular fin, $P = 2(w + t) \simeq 2w$ for $w \gg t$
  - Profile area, $A_p = L_c t$
  - $$mL_c = \sqrt{\frac{h_c P}{k A_c}} L_c = \sqrt{\frac{2h}{kt}} L_c = \sqrt{\frac{2h}{kA_p}} L_c^{3/2}$$



T682

Efficiency of rectangular and triangular fins.

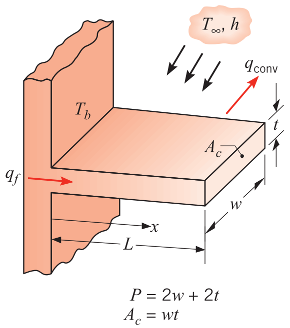


T683

Efficiency of circumferential rectangular fins.

Example: ▷ An aluminum fin [k =200 W/m°C], 3.0 mm thick and 7.5 cm long protrudes from a wall. The base is maintained at 300°C, and T_∞ = 50°C with h=10 W/m^2°C. Calculate the heat loss from the fin per unit depth of material.

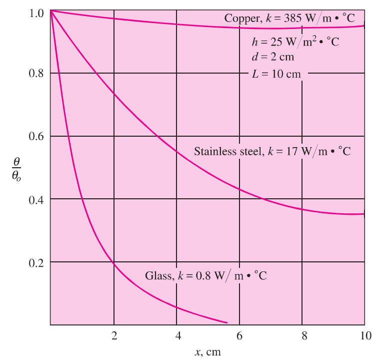
[a) Case 4: 360.42 W, b) Case 2 with L_c: 360.44 W.]



T699

Example: ▷ Compare the temperature distributions in a straight cylindrical rod having a diameter of 2 cm and a length of 10 cm and exposed to a convection environment with h=25 W/m2 °C, for three fin materials: copper [k =385 W/m°C], stainless steel [k =17 W/m°C], and glass [k =0.8 W/m°C]. Also compare the relative heat flows and fin efficiencies.

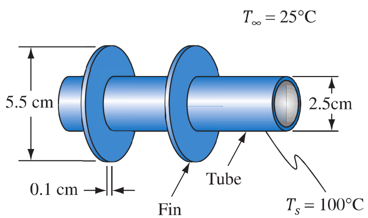
[η_{copper} = 0.9549, η_{steel} = 0.5258, η_{glass} = 0.1205]



T686

Example: ▷ Circumferential fins made of aluminium (k=200 W/m K) are soldered to the outer surface, and h_c= 65 W/m^2 K, calculate the rate of heat loss from a fin.

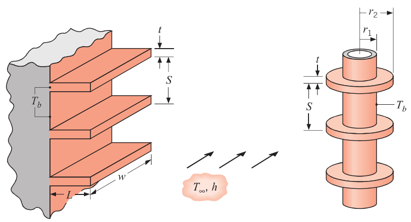
[17.8 W]



T695

Fin: Overall Surface Efficiency

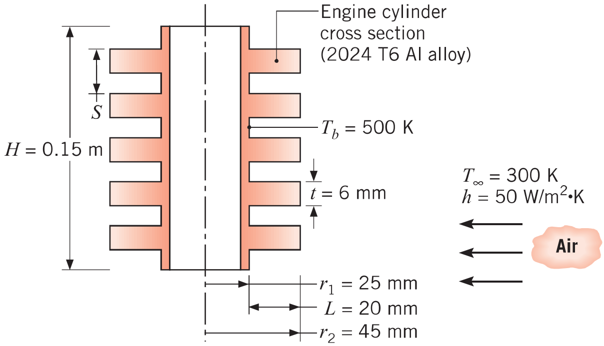
In contrast to the fin efficiency, η_f, which characterizes the performance of a single fin, the overall surface efficiency, η_o characterizes an array of fins and the base surface to which they are attached.



T694

- $\eta_o = \frac{q_t}{q_{max}} = \frac{q_t}{h_c A_t \theta_s}$
- Total surface area, $A_t = NA_f + A_b$, $N \equiv$ fins in array with area A_f
- $q_t = N\eta_f hA_f\theta_s + h_c A_b \theta_s = h_c A_t \left[1 - \frac{NA_f}{A_t}(1-\eta_f)\right]\theta_s$
- $\eta_o = 1 - \frac{NA_f}{A_t}(1-\eta_f)$

Example: ▷ Motorcycle engine cylinder: estimate the increase in heat transfer due to fins. [460 W]

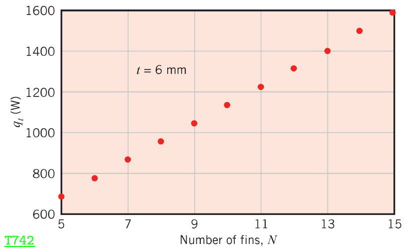


T679

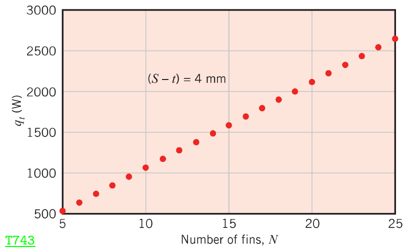
If gap of 4 mm between the fins is maintained:

- If t = 6 mm:
- $N_{max} = H/(0.004+0.006) = 15$

- If t = 2 mm:
- $N_{max} = H/(0.004+0.002) = 25$



T742

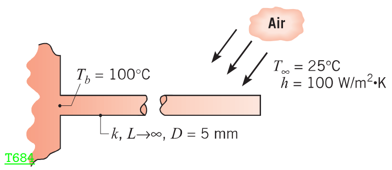


T743

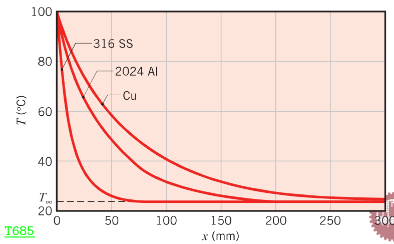
HW Problem:

Example: ▷

1. Determine the temperature distributions along rods constructed from pure copper, 2024 aluminum alloy, and type AISI 316 stainless steel. What are the corresponding heat losses from the rods?
2. Estimate how long the rods must be for the assumption of infinite length to yield an accurate estimate of the heat loss



T684



T685