HX: Effective NTU Method

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ME 307: Heat Transfer Equipment Design

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ϵ-NTU Method

- If the inlet & outlet temperatures of the hot & cold fluid, and the overall heat transfer coefficients are specified, the LMTD method can be used to solve rating and sizing problem (e.g. process, power and petrochemical industries).
- If only the inlet temperatures and fluid (hot & cold) flow rates are given LMDT estimation requires cumbersome iterations.
- ϵ-NTU method is generally performed for the design of compact heat exchangers for automotive, aircraft, air conditioning, and various industrial applications where the inlet temperatures of the hot and cold fluids are specified and the heat transfer rates are to be determined.

Holman Ex. 10-8 ▷ A cross-flow heat exchanger, one fluid mixed and one unmixed, is used to heat oil at 15°C in the tubes (1.45 kg/s, c = 1.9 kJ/kg). Steam (5.2 kg/s, $c = 1.86$ kJ/kg°C) blows across the outside of the tube, enters at 130°C. $U_o = 275$ W/m^2 °C . Calculate A.

- $\dot{m}_h = 5.2$ kg/s, $c_{p,h} = 1.86$ kJ/kg, $T_{h,i} = 130$°C.
- $\dot{m}_c = 1.45$ kg/s, $c_{p,c} = 1.9$ kJ/kg, $T_{c,i} = 15$°C.
- $\dot{q} = \dot{m}_h c_{p,h}(T_{h,i} - T_{h,o}) = \dot{m}_c c_{p,c}(T_{c,o} - T_{c,i})$
- $\dot{q} = UA\Delta T_{LM}$

⇒ complex iteration is required to get $T_{c,o}$ and $T_{h,o}$.

Maximum Heat Transfer Rate, q_{max}

- Maximum HT could be achieved in counter-flow HX with $x \to \infty$.
- If $C_c < C_h$: $|\Delta T_c| > |\Delta T_h|$, COLD fluid have ΔT_{max}.

⇒ If $x \to \infty \Rightarrow T_{c,o} = T_{h,i} \Rrightarrow \dot{q}_{max} = C_c(T_{c,o} - T_{c,i}) = C_c(T_{h,i} - T_{c,i})$

- If $C_c > C_h$: $|\Delta T_c| < |\Delta T_h|$, HOT fluid have ΔT_{max}.

⇒ If $x \to \infty \Rightarrow T_{h,o} = T_{c,i} \Rrightarrow \dot{q}_{max} = C_h(T_{h,i} - T_{h,o}) = C_h(T_{h,i} - T_{c,i})$

⇒ $\boxed{\dot{q}_{max} = C_{min}(T_{h,i} - T_{c,i}) = C_{min}\Delta T_{max}}$

⇒ $\boxed{\text{Effectiveness, } \epsilon \equiv \frac{\dot{q}}{\dot{q}_{max}} = \frac{C_h(T_{h,i} - T_{h,o})}{C_{min}(T_{h,i} - T_{c,i})} \text{ or } \frac{C_c(T_{c,o} - T_{c,i})}{C_{min}(T_{h,i} - T_{c,i})}}$

$\dot{q} \equiv$ actual rate of heat transfer,

$\dot{q}_{max} \equiv$ maximum possible rate of heat transfer with some inlet temperatures, flow rates and specific heats as actual case.

ϵ-NTU Method

- Number of Transfer Unit, $\boxed{NTU \equiv \frac{UA}{C_{min}}}$
- Capacity ratio, $\boxed{C_R \equiv \frac{C_{min}}{C_{max}}}$
- The value of capacity ratio, C_R ranges between 0 and 1.0.
- $C_R = C_{min}/C_{max} \to 0$ corresponds to $C_{max} \to \infty$, which is realized during a phase-change process in a condenser or boiler.
- Effectiveness is lowest for $C_R = C_{min}/C_{max} = 1$, when heat rates of two fluids are the same. Example, HVAC applications.
- $\epsilon = f(NTU, C_R, HX\ type)$
- $\dot{q}_{max} = C_{min}(T_{h,i} - T_{c,i}) = C_{min}\Delta T_{max}$
- $\dot{q} = \epsilon\ \dot{q}_{max}$
- $T_{h,o} = T_{h,i} - \frac{\dot{q}}{C_h}$
- $T_{c,o} = T_{c,i} + \frac{\dot{q}}{C_c}$

ϵ-NTU Method: Parallel-Flow Heat Exchanger

- $\frac{d(\Delta T)}{\Delta T} = -U\left[\frac{1}{C_h} + \frac{1}{C_c}\right] dA \to \ln(\Delta T_2/\Delta T_1) = -UA\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$

$\Rightarrow \frac{\Delta T_2}{\Delta T_1} = \frac{T_{h,o} - T_{c,o}}{T_{h,i} - T_{c,i}} = \exp(-NTU(1 + C_R))$

- If $C_{min} = C_h$:, $C_R = \frac{C_h}{C_c} = \frac{\dot{m}_h c_{p,h}}{\dot{m}_c c_{p,c}} = \frac{T_{c,o} - T_{c,i}}{T_{h,i} - T_{h,o}}$

$\Rightarrow \epsilon = \frac{C_c(T_{c,o} - T_{c,i})}{C_{min}(T_{h,i} - T_{c,i})} = \frac{1}{C_R}\frac{T_{c,o} - T_{c,i}}{T_{h,i} - T_{c,i}} = \frac{T_{h,i} - T_{h,o}}{T_{h,i} - T_{c,i}}$

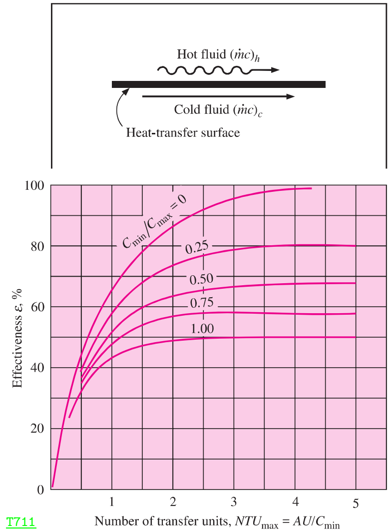
$\Rightarrow \frac{T_{h,o} - T_{c,o}}{T_{h,i} - T_{c,i}} = -\epsilon + 1 - C_R\epsilon \to \epsilon = \frac{1 - \exp(-NTU(1 + C_R))}{1 + C_R}$

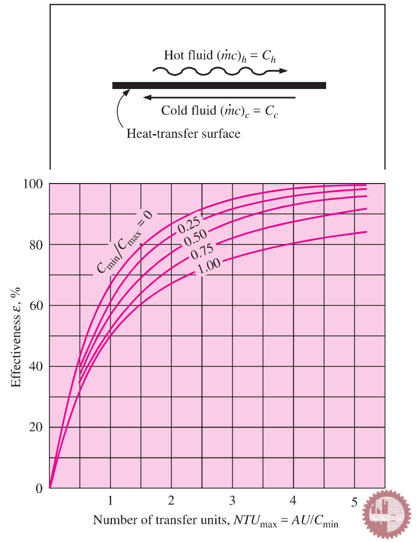
- Same result is obtained, if $C_{min} = C_c$

$\Rightarrow \epsilon = \frac{1 - \exp(-NTU(1 + C_R))}{1 + C_R}$: Parallel-Flow HX

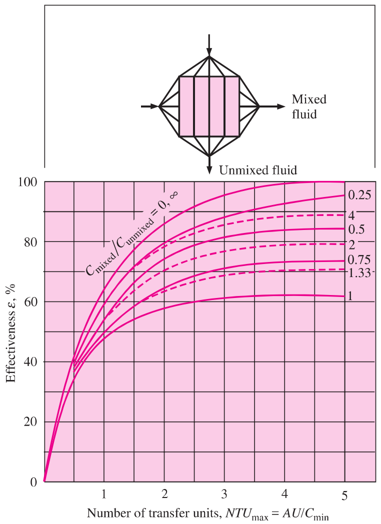
One fluid (or any configuration with $C_R = 0$)		$\varepsilon = 1 - \exp(-NTU)$
Counter-flow		$\varepsilon = \begin{cases} \frac{1 - \exp[-NTU(1 - C_R)]}{1 - C_R\exp[-NTU(1 - C_R)]} & \text{for } C_R < 1 \\ \frac{NTU}{1 + NTU} & \text{for } C_R = 1 \end{cases}$
Parallel-flow		$\varepsilon = \frac{1 - \exp[-NTU(1 + C_R)]}{1 + C_R}$
Cross-flow	both fluids unmixed	$\varepsilon = 1 - \exp\left[\frac{NTU^{0.22}}{C_R}\{\exp(-C_R\,NTU^{0.78}) - 1\}\right]$
	both fluids mixed	$\varepsilon = \left[\frac{1}{1 - \exp(-NTU)} + \frac{C_R}{1 - \exp(-C_R\,NTU)} - \frac{1}{NTU}\right]^{-1}$
	$\dot{C}_{max}$ mixed & $\dot{C}_{min}$ unmixed	$\varepsilon = \frac{1 - \exp[C_R\{\exp(-NTU) - 1\}]}{C_R}$
	$\dot{C}_{min}$ mixed & $\dot{C}_{max}$ unmixed	$\varepsilon = 1 - \exp\left[-\frac{1 - \exp(-C_R\,NTU)}{C_R}\right]$
Shell-and-tube	one shell pass & an even # of tube-passes	$\varepsilon_1 = 2\left[1 + C_R + \sqrt{1 + C_R^2}\frac{1 + \exp\left(-NTU_1\sqrt{1 + C_R^2}\right)}{1 - \exp\left(-NTU_1\sqrt{1 + C_R^2}\right)}\right]^{-1}$
	N shell passes & 2N, 4N, . . . tube-passes	$\varepsilon = \frac{\left(\frac{1 - \varepsilon_1 C_R}{1 - \varepsilon_1}\right)^N - 1}{\left(\frac{1 - \varepsilon_1 C_R}{1 - \varepsilon_1}\right)^N - C_R}$ where ε_1 and NTU_1 is for one shell pass

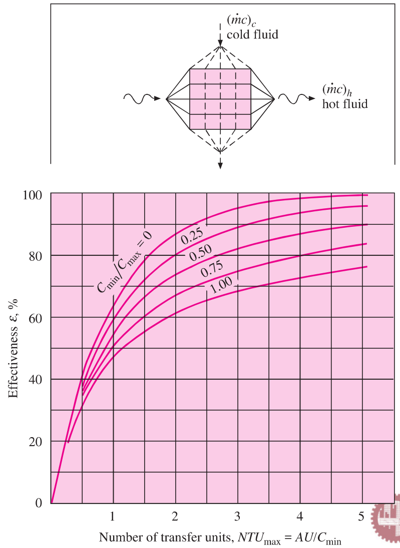
T710



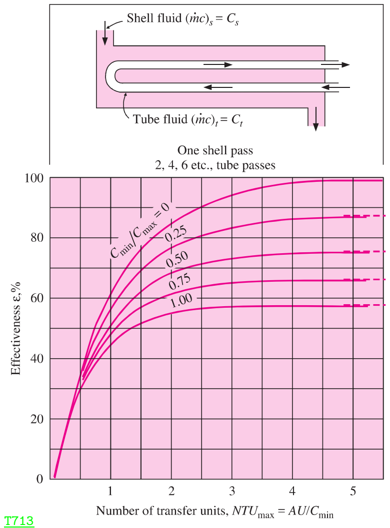


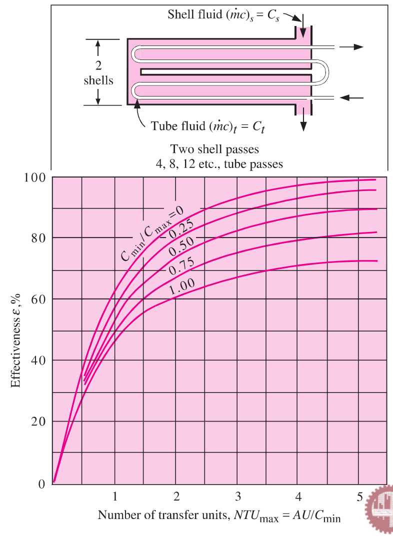
T711





T712 C_{max} unmixed $\longrightarrow$ solid



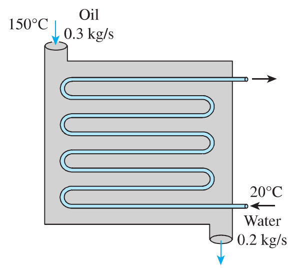


T713

Holman Ex. 10-10 ▷ In a counterflow double-pipe heat exchanger (15.82 m^2), water at the rate of 40 kg/min is heated from 35°C by an oil having a specific heat of 1.9 kJ/kg°C entering at 110°C at a rate of 171 kg/min. Given that, U_o = 320 W/m^2°C. Estimate heat transfer and the fluid outlet temperatures.

Holman Ex. 10-9 ▷ A cross-flow heat exchanger, one fluid mixed and one unmixed, is used to heat an oil in the tubes (c = 1.9 kJ/kg °C, 15°C). Steam (5.2 kg/s, c = 1.86 kJ/kg °C, 130°C) blows across the outside of the tube. If oil flow rate is 0.725 kg/s, U_o = 275 W/m^2°C, A = 10.82 m^2, rate the HX.

Cengel Ex. 11-9 ▷ Hot oil (c = 2.13 kJ/kgK) is to be cooled by water in a 1-shell-pass and 8-tube-passes heat exchanger. The tubes are thin-walled and are made of copper with an internal diameter of 1.4 cm. The length of each tube pass in the heat exchanger is 5 m, and the overall heat transfer coefficient is 310 W/m^2K. Rate the HX.



T1004

Ozisik Ex. 11-15 ▷ A two shell pass, four tube pass heat exchanger is used to cool oil (cp = 2100 J/kgK) at 1.5 kg/s from 90°C to 40°C with water entering at 19°C and at 1.0 kg/s. If U = 250 W/m^{2}K, estimate the heat transfer area required.

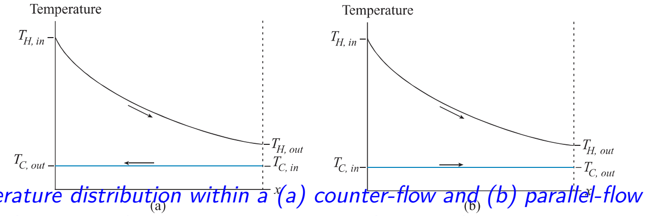
Ozisik Ex. 11-16 ▷ A shell-and-tube steam condenser is constructed at 2.5 cm-OD, single pass horizontal tubes with steam at 1.5 kg/s condensing at 54°C. The cooling water enters the tube at 18°C with a flow rate of 0.7 kg/s and leaves at 36°C. If U = 3509 W/m^{2}K, estimate the heat transfer rate and tube length required.

HX: Behaviour as $C_R \longrightarrow 0$

- $C_R \longrightarrow 0$: one fluid stream has a capacity rate which is much higher than the other fluid stream. For all configurations,

$$\lim_{C_R \to 0} \epsilon = 1 - \exp(-NTU)$$

- Examples: interaction of a flowing fluid with a constant temperature solid (as in a cold-plate) or a well-mixed tank of fluid or the situation that occurs when one of the two streams is undergoing constant pressure evaporation or condensation.



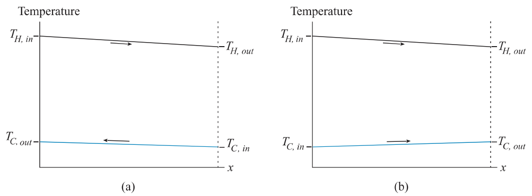
T779

Temperature distribution within a (a) counter-flow and (b) parallel-flow heat exchanger as the capacity ratio approaches zero because $C_c >> C_h$.

HX: Behaviour as $NTU \longrightarrow 0$: Small HTX

- If NTU is small, heat exchanger is under-sized & rate of heat transfer is not sufficient to change the temperature of either fluid substantially. Hence, $\Delta T = (T_{h,in} - T_{c,out})$
- $NTU \longrightarrow 0$: $q = UA(T_{H,in} - T_{C,in})$, $\epsilon \equiv \frac{q}{C_{min}(T_{H,in} - T_{C,in})}$
- For all configurations,

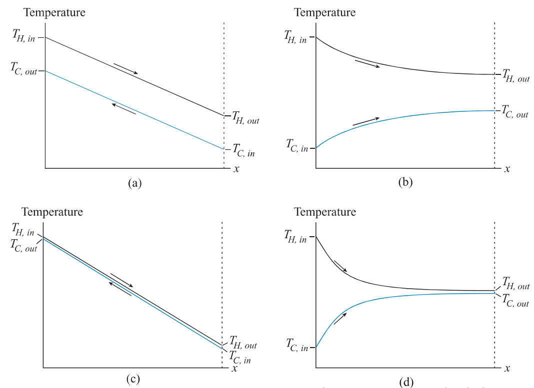
$$\lim_{NTU \to 0} \epsilon = NTU$$



T780

Temperature distribution within a (a) counter-flow and (b) parallel-flow heat exchanger as NTU approaches zero.

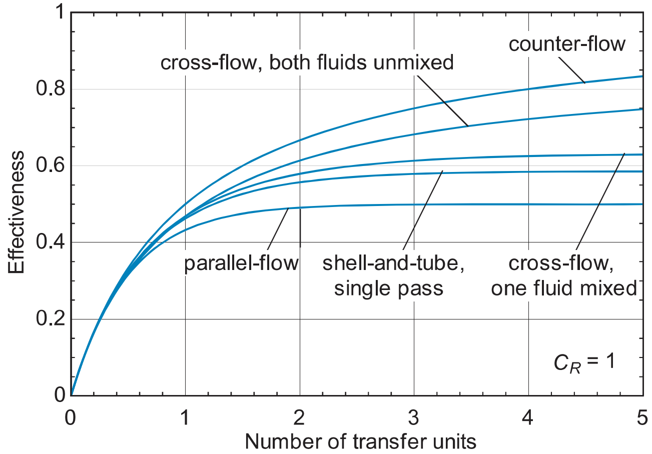
HX: Behaviour as $NTU \longrightarrow \infty$: Very Large HTX



T781

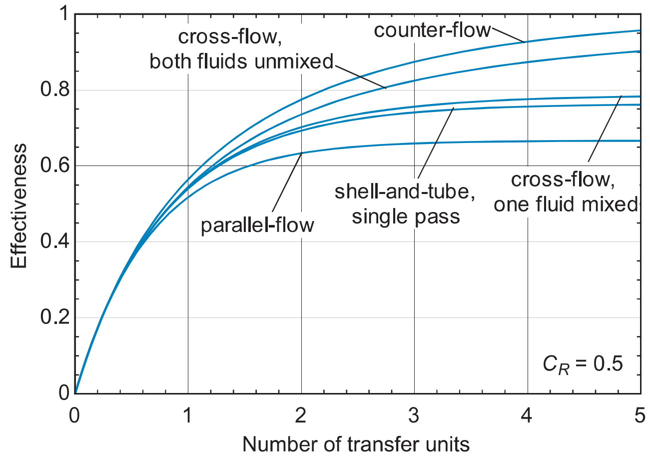
Temperature distribution within a balanced (i.e., $C_R = 1$) (a) counter-flow and (b) parallel-flow heat exchanger for a finite NTU. The temperature distribution within a balanced (c) counter-flow and (d) parallel-flow heat exchanger as $NTU \longrightarrow \infty$.

Effectiveness as a function of *NTU*



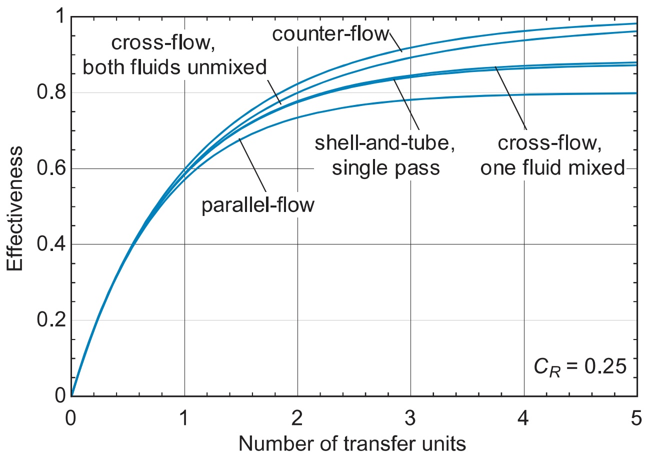
T782

ϵ as a function of NTU for various configurations ($C_R = 1$).



T783

ϵ as a function of NTU for various configurations ($C_R = 0.5$).



T784

ϵ as a function of NTU for various configurations ($C_R = 0.25$).