HX: Energy Balance and LMTD Method

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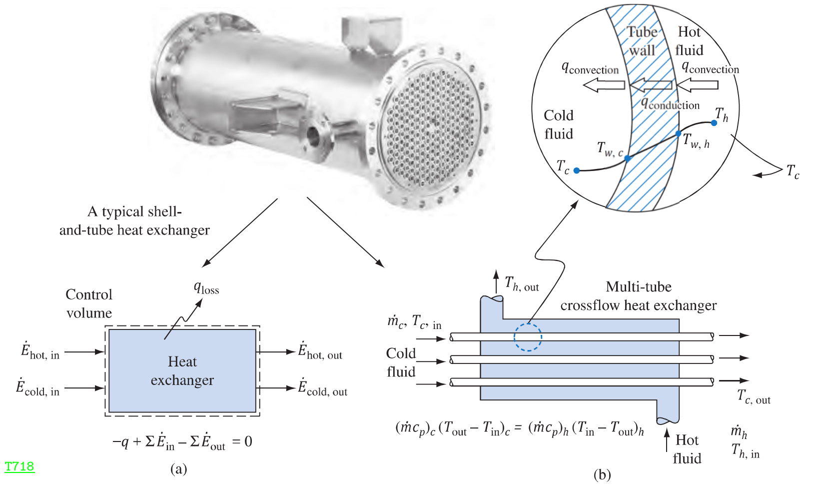
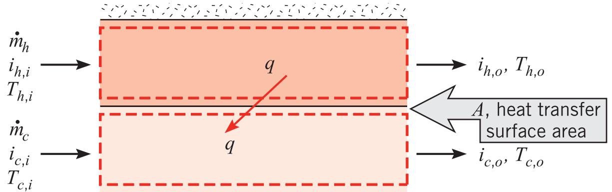
ME 307: Heat Transfer Equipment Design

`http://zahurul.buet.ac.bd/ME307/`



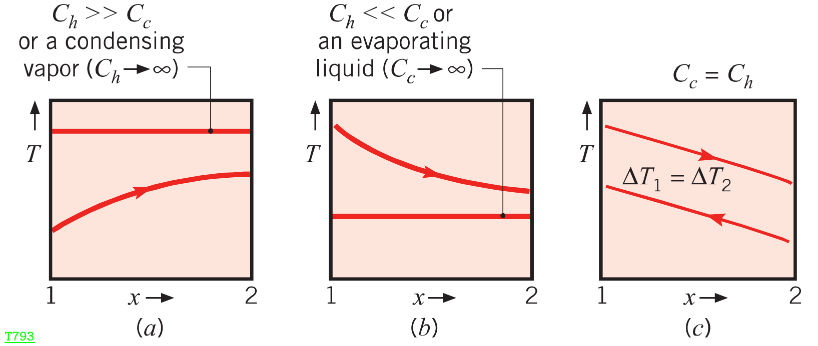
Application of and contrast between (a) a thermodynamic and (b) a heat transfer model for a typical shell-and-tube heat exchanger used in chemical processing.

Heat Exchanger: Energy Balance Equation



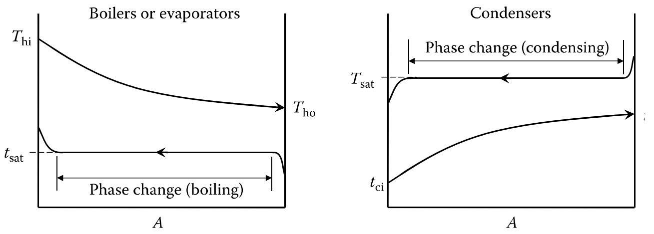
- $\boxed{\dot{q} = \dot{m}_h(i_{h,i} - i_{h,o}) = \dot{m}_h c_{p,h}(T_{h,i} - T_{h,o}) = C_h(T_{h,i} - T_{h,o})}$
- $\boxed{\dot{q} = \dot{m}_c(i_{c,o} - i_{c,i}) = \dot{m}_c c_{p,c}(T_{c,o} - T_{c,i}) = C_c(T_{c,o} - T_{c,i})}$
- $\boxed{\dot{q} = UA\Delta T_m}$
- i is fluid enthalpy, subscripts h & c refer to hot & cold fluids, whereas the subscripts i & o designate fluid inlet & outlet conditions.
- $C \equiv \dot{m}c_p \equiv$ heat capacity rate [W/K]
- $U \equiv$ overall heat transfer coefficient [W/m^2 K]
- ΔT_m is an appropriate mean temperature difference.

Special Heat Exchanger Conditions



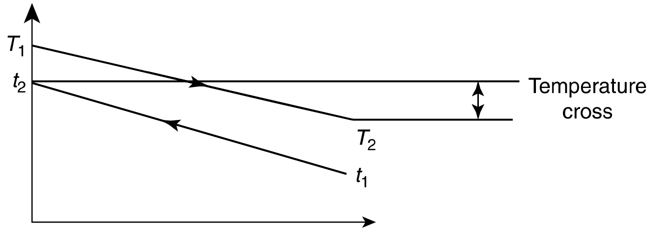
(a) $C_h >> C_c$ or a condensing vapour. (b) An evaporating liquid or $C_h << C_c$. (c) A counterflow heat exchanger with equivalent fluid heat capacities ($C_h = C_c$).

A condenser or a boiler can be considered to be either a parallel or counterflow HTX since both approaches give the same result.



T1029

Typical temperature profiles inside boilers, evaporators, and condensers.



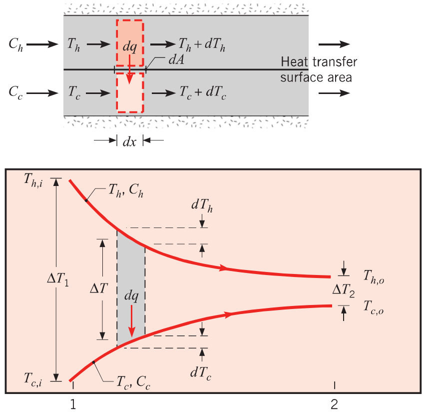
T1030

Temperature cross in a counterflow heat exchanger.

LMTD: Log Mean Temperature Difference, ΔT_{LM}

- $\dot{q} = UA\Delta T_{LM}$
- $\dot{q} = -\dot{m}_h c_{p,h}\Delta T_h = -C_h\Delta T_h$
- $\dot{q} = +\dot{m}_c c_{p,c}\Delta T_c = +C_c\Delta T_c$
- $d\dot{q} = -C_h dT_h = +C_c dT_c$
- $d\dot{q} = U(T_h - T_c)dA = U\Delta T dA$
- $\Delta T \equiv T_h - T_c$
- $d\Delta T = dT_h - dT_c = -\left[\frac{1}{C_h} + \frac{1}{C_c}\right] d\dot{q} = -\left[\frac{1}{C_h} + \frac{1}{C_c}\right] U\Delta T dA$
- $\frac{d\Delta T}{\Delta T} = -U\left[\frac{1}{C_h} + \frac{1}{C_c}\right] dA \longmapsto \ln\left[\frac{\Delta T_2}{\Delta T_1}\right] = -U\left[\frac{1}{C_h} + \frac{1}{C_c}\right] A$
- $\ln\left[\frac{\Delta T_2}{\Delta T_1}\right] = \frac{UA}{\dot{q}}[\Delta T_h - \Delta T_c]$

LMTD: Parallel-Flow Heat Exchanger

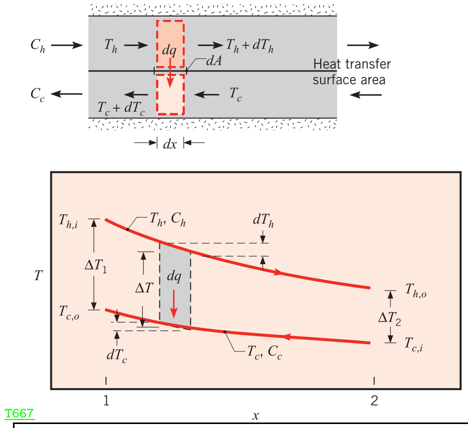


T702

- $\Delta T_1 = (T_{h,i} - T_{c,i})$
- $\Delta T_2 = (T_{h,o} - T_{c,o})$
- $\Delta T_h - \Delta T_c = \Delta T_2 - \Delta T_1$

$$\dot{q} = UA\left[\frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2/\Delta T_1)}\right] = UA\Delta T_{LM}$$

LMTD: Counter-Flow Heat Exchanger



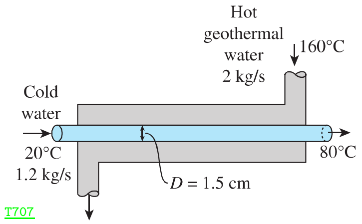
T667

- $\Delta T_1 = (T_{h,i} - T_{c,o})$
- $\Delta T_2 = (T_{h,o} - T_{c,i})$
- $\Delta T_h - \Delta T_c = \Delta T_2 - \Delta T_1$

$$\dot{q} = UA\left[\frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2/\Delta T_1)}\right] = UA\Delta T_{LM}$$

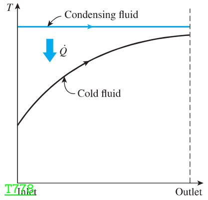
Cengel Ex. 11-4 ▷ Heating Water in a Counter-Flow Heat Exchanger: If U_o = 640 W/m^{2}K, determine the length of the heat exchanger required to achieve the desired heating. Re-estimate the length for Parallel-Flow configuration.

[109 m, 120 m]

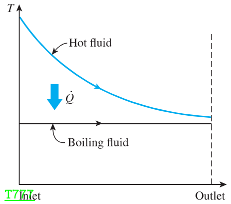


T707

Condenser, Evaporator (Boiler)



T778

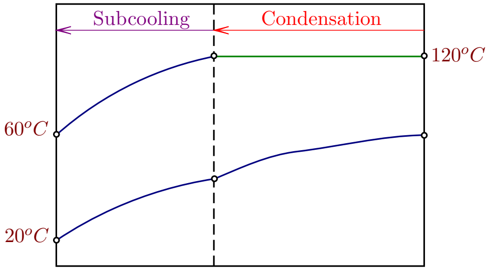


T777

- **Condenser**: $Q = \dot{m}_h(i_i - i_o) = \dot{m}_h \Delta x_i i_{fg}$
- $Q = (\dot{m}C_p)_c(T_{co} - T_{ci}) = UA\Delta T_{LM}$
- $\Delta T_{LM} = \frac{(T_{sat}-T_{ci})-(T_{sat}-T_{co})}{\ln\left(\frac{T_{sat}-T_{ci}}{T_{sat}-T_{co}}\right)} = \frac{T_{co}-T_{ci}}{\ln\left(\frac{T_{sat}-T_{ci}}{T_{sat}-T_{co}}\right)}$
- $A = \frac{C_c}{U}\ln\left(\frac{T_{sat}-T_{ci}}{T_{sat}-T_{co}}\right) \longrightarrow \boxed{\frac{T_{sat}-T_{ci}}{T_{sat}-T_{co}} = \exp\left(-\frac{UA}{C_c}\right)}$
  - **Boiler**: $Q = \dot{m}_c(i_o - i_i) = \dot{m}_c \Delta x_i i_{fg}$
  - $Q = (\dot{m}C_p)_h(T_{hi} - T_{ho}) = UA\Delta T_{LM}$
  - $\Delta T_{LM} = \frac{(T_{hi}-T_{sat})-(T_{ho}-T_{sat})}{\ln\left(\frac{T_{hi}-T_{sat}}{T_{ho}-T_{sat}}\right)} = \frac{T_{hi}-T_{ho}}{\ln\left(\frac{T_{hi}-T_{sat}}{T_{ho}-T_{sat}}\right)}$
  - $A = \frac{C_h}{U}\ln\left(\frac{T_{hi}-T_{sat}}{T_{ho}-T_{sat}}\right) \longrightarrow$ $\boxed{\frac{T_{hi}-T_{sat}}{T_{ho}-T_{sat}} = \exp\left(-\frac{UA}{C_h}\right)}$

▶ Δx change of dryness of condensing/boiling fluid.

Condenser Sizing: ▷ Saturated water at 120°C with a quality of $x_i = 0.2$ and a mass flow rate, $\dot{m}_h$ = 10 kg/s is to be cooled to 60°C with a water flow of $\dot{m}_c$ = 40 kg/s at 20°C. If for liquid water, h_w = 8000 W/m^{2}K, and for condensing vapour, h_v = 24000 W/m^{2}K, and C_p = 4200 J/KgK for liquid water, and h_{fg} = 2200 kJ/kg at 120°C, estimate the heat transfer surface area. [21 m^2]



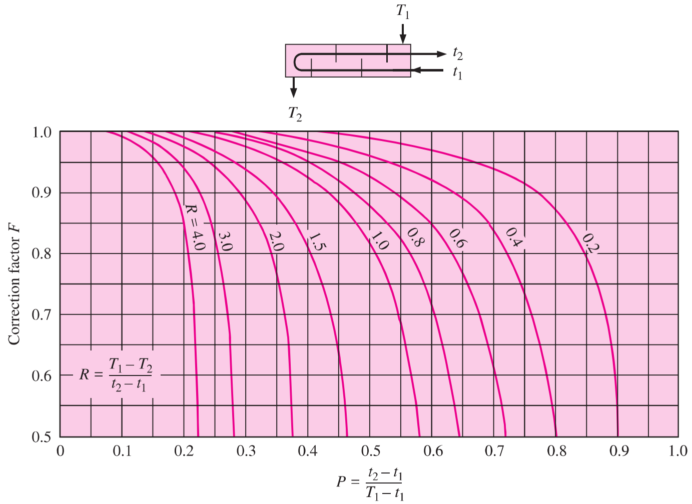
T769

Correction for LMDT: Cross-Flow & Multipass HTX

- If a heat exchanger other than the double-pipe type is used, $\dot{q}$ is calculated by using a correction factor, F applied to LMDT for counter-flow arrangement with same hot and cold fluid temperatures.

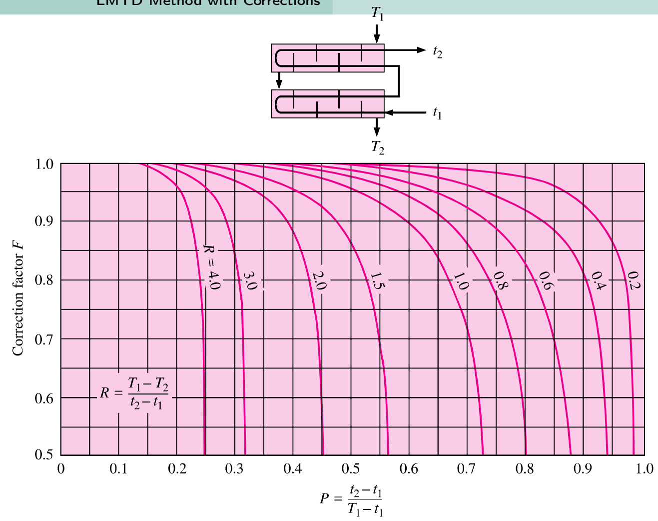
$$\boxed{\dot{q} = F\ UA\Delta T_{LM}}$$

- F = 1.0 for boiling and condensation.
- F-values are presented in figures, using two dimensionless parameters:
  - ① $P \equiv \frac{t_2-t_1}{T_1-t_1}$: $0 \leqslant P \leqslant 1.0$: thermal efficiencies of tube-side flow.
  - ② $R \equiv \frac{T_1-T_2}{t_2-t_1} \cong \frac{(C)_{tube-side}}{(C)_{shell-side}}$: $0 \leqslant R \leqslant \infty$:
    - $R \rightarrow 0$: vapour condensation on the shell side
    - $R \rightarrow \infty$: evaporation on the tube side.
    - $T \equiv$ shell-side temperatures
    - $t \equiv$ tube-side temperatures



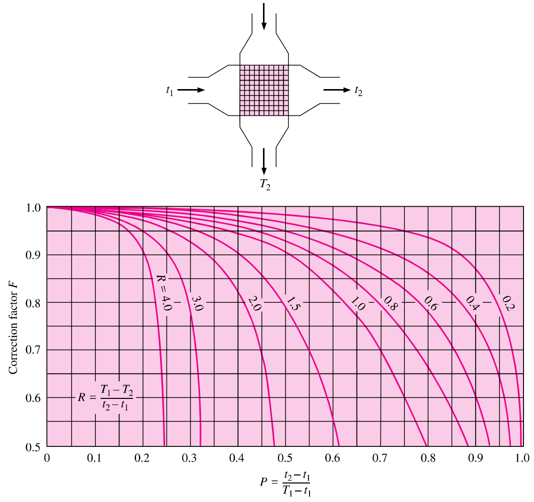
T714

Correction-factor, F for exchanger with one shell pass and two, four, or any multiple of tube passes.



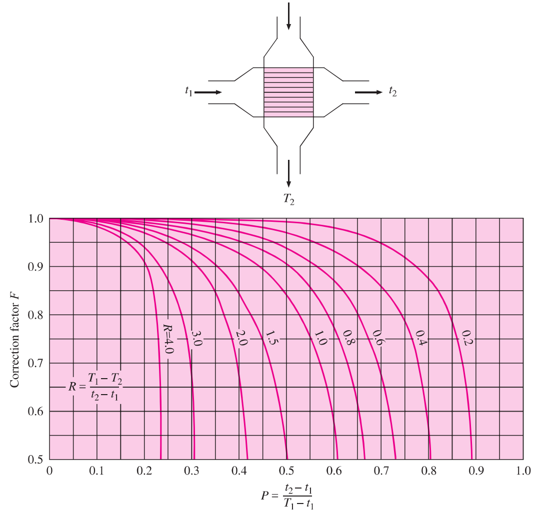
T715

Correction-factor, F for exchanger with two shell passes and four, eight, or any multiple of tube passes.



T716

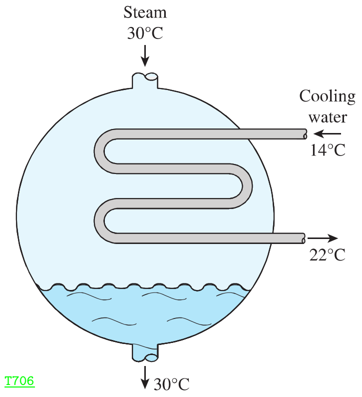
Correction-factor, F for single-pass cross-flow exchanger, both fluids unmixed.



T717

Correction-factor, F for single-pass cross-flow exchanger, one fluid mixed, the other unmixed.

Cengel Ex. 11-3 ▷ Steam condenser with surface area of the tubes is 45 m^2, and the overall heat transfer coefficient is 2100 W/m^{2}K. Estimate condenser capacity. [1.09 MW]



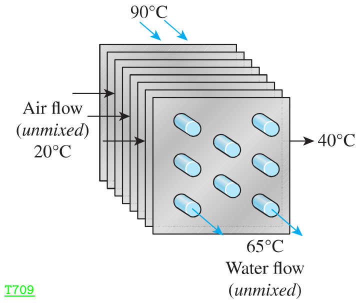
T706

Holman Ex. 10-4 ▷ In a counterflow double-pipe heat exchanger, water at the rate of 68 kg/min is heated from 35 to 75°C by an oil having a specific heat of 1.9 kJ/kg°C. Oil enters the exchanger at 110°C and leaves at 75°C. Given that, U_o = 320 W/m^2°C. Estimate heat transfer area, A. [15.8 m^2]

Holman Ex. 10-5 ▷ Instead of the double-pipe heat exchanger of Ex. 10-4, it is desired to use a shell-and-tube exchanger with the water making one shell pass and the oil making two tube passes. Calculate the area, A, assuming that the overall heat-transfer coefficient remains at 320 W/m^2°C. Recalculate with the fluid swapping. [18.6 m^2]

Holman Ex. 10-7 ▷ A cross-flow heat exchanger, one fluid mixed and one unmixed, is used to heat an oil in the tubes (c = 1.9 kJ/kg °C) from 15°C to 85°C. Steam (5.2 kg/s, c = 1.86 kJ/kg°C) blows across the outside of the tube, enters at 130°C and leaves at 110°C. U_o = 275 W/m^2 °C . Calculate A. [10.8 m^2]

Cengel Ex. 11-6 ▷ Cooling of Water in an Automotive Radiator: The radiator has 40 tubes of internal diameter 0.5 cm and length 65 cm in a closely spaced plate-finned matrix. Hot water enters the tubes at a rate of 0.6 kg/s. Determine the overall heat transfer coefficient U_i of this radiator based on the inner surface area of the tubes. [U = 3335 W/m^2 K]



T709

Ozisik Ex. 11-12 ▷ A heat exchanger is to be designed to cool 8.7 kg/s an ethyl alcohol solution [$c_{p,h}$ = 3840 J/kg°C] from 75°C to 45°C with cooling water [$c_{p,c}$ = 4180 J/kg°C entering the tube side at 15°C at a rate of 9.6 kg/s. Given, U_o = 500 W/m^2°C. Estimate heat transfer area, for:

❶ parallel flow DPHX

❷ counter flow DPHX

❸ one shell pass and two tube pass, STHX

❹ cross-flow, both fluids unmixed, CFHX

❺ cross-flow, one fluid unmixed, CFHX