Second Law Analysis for a Control Volume

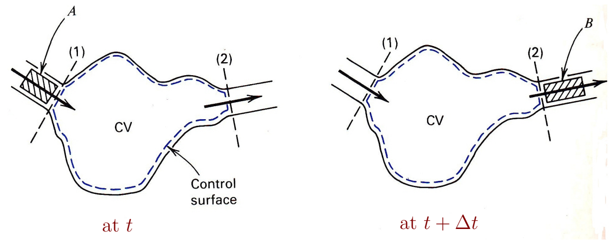
Dr. Md. Zahurul Haq, *Ph.D., CEA, FBSME, FIEB*

Professor
Department of Mechanical Engineering
Bangladesh University of Engineering & Technology (BUET)
Dhaka-1000, Bangladesh

zahurul@me.buet.ac.bd
http://zahurul.buet.ac.bd/

ME 203: Engineering Thermodynamics
`http://zahurul.buet.ac.bd/ME203/`

Second Law Analysis for CV System



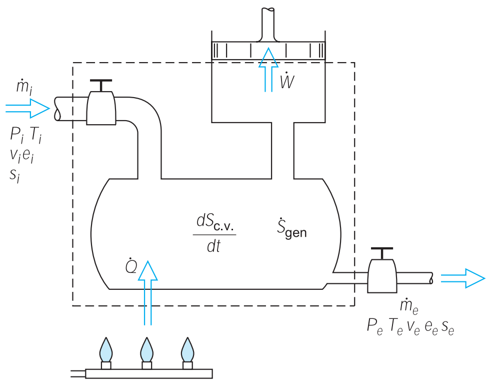
T320

CM (time t) : region A + CV $\quad$ CM ($t + \Delta t$) : CV + region B

$S_{cm,t} = S_A + S_{cv,t}$ $\quad$ $S_{cm,t+\Delta t} = S_B + S_{cv,t+\Delta t}$

- $\frac{S_{cm,t+\Delta t} - S_{cm,t}}{\Delta t} = \frac{S_{cv,t+\Delta t} - S_{cv,t}}{\Delta t} + \frac{S_B - S_A}{\Delta t} :\Rightarrow \frac{dS_{cm}}{dt} = \frac{dS_{cv}}{dt} + \dot{m}_B s_B - \dot{m}_A s_A$
- $\frac{dS_{cm}}{dt} = \sum \frac{\dot{Q}_i}{T_i} + \dot{\sigma}$

$$\frac{dS_{cv}}{dt} = \sum \frac{\dot{Q}_i}{T_i} + \sum_i (\dot{m}s)_i - \sum_e (\dot{m}s)_e + \dot{\sigma}_{cv}$$



T042

$$\underbrace{\frac{dS_{cv}}{dt}}_{\text{rate of entropy change}} = \underbrace{\sum_j \frac{\dot{Q}_j}{T_j} + \sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e}_{\text{rates of entropy transfer}} + \underbrace{\dot{\sigma}_{cv}}_{\text{rate of entropy production}}$$

T175

$$\frac{dS_{cv}}{dt} = \sum_j \frac{\dot{Q}_j}{T_j} + \sum (\dot{m}s)_i - \sum (\dot{m}s)_e + \dot{\sigma}_{cv}$$

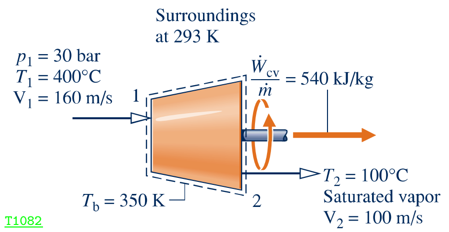
- For CM systems: $\dot{m}_i = 0$, $\dot{m}_e = 0 \Rightarrow$ $\boxed{\frac{dS_{cm}}{dt} = \sum_j \frac{\dot{Q}_j}{T_j} + \dot{\sigma}}$
- For steady-state steady-flow (SSSF) process: $dS_{dv}/dt = 0$.
- For 1-inlet & 1-outlet SSSF process: $\dot{m}_i = \dot{m}_e$.

$\Rightarrow$ $$(s_e - s_i) = \sum_j \frac{\dot{q}_j}{T_j} + \frac{\dot{\sigma}_{cv}}{\dot{m}}$$

- For adiabatic 1-inlet & 1-outlet SSSF process:

$\Rightarrow$ $$(s_e - s_i) = \frac{\dot{\sigma}_{cv}}{\dot{m}} \Rightarrow s_e \geqq s_i$$

Moran Ex. 6-6 ▷ Determine the rate at which entropy is produced within the turbine per kg of steam flowing, in kJ/kgK.



T1082

- SSSF: $\frac{dS_{CV}}{dt} = 0$, $\dot{m}_i = \dot{m}_e = \dot{m}$
- $\sum \frac{\dot{Q}_i}{T_i} = \frac{\dot{Q}}{T_b}$: $T_b = 350$ K.
- $z_i = z_e$
- States ① & ②: defined.

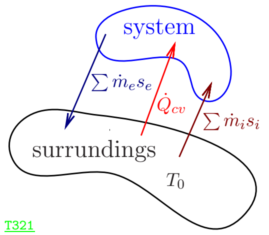
- $\frac{dE_{CV}}{dt} = \dot{Q} - \dot{W}_{cv} + \dot{m}_i \left(h_i + \frac{V_i^2}{2} + gz_i\right) - \dot{m}_e \left(h_e + \frac{V_e^2}{2} + gz_e\right)$

$\Rightarrow$ $\dot{Q} = \surd$

- $\frac{dS_{CV}}{dt} = \sum \frac{\dot{Q}_i}{T_i} + (\dot{m}s)_i - (\dot{m}s)_e + \dot{\sigma}_{CV}$

$\Rightarrow$ $\dot{\sigma}_{CV} = \surd$

Principle of Increase of Entropy



T321

- $\frac{dS_{cv}}{dt} = \sum \frac{\dot{Q}_i}{T_i} + \sum_i (\dot{m}s)_i - \sum_e (\dot{m}s)_e + \dot{\sigma}_{cv}$

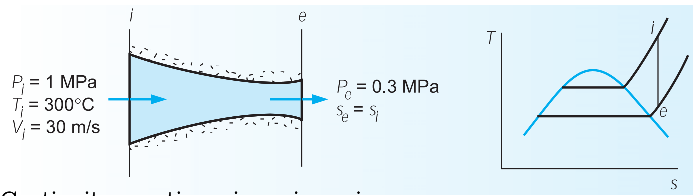
$\Rightarrow$ $\frac{dS_{cv}}{dt} = \frac{\dot{Q}_{cv}}{T_i} + \sum_i (\dot{m}s)_i - \sum_e (\dot{m}s)_e + \dot{\sigma}_{cv}$

- $\frac{dS_{surr}}{dt} = \frac{\dot{Q}_i}{T_i} + \sum_i (\dot{m}s)_i - \sum_e (\dot{m}s)_e + \dot{\sigma}_{surr}$

$\Rightarrow$ $\frac{dS_{surr}}{dt} = -\frac{\dot{Q}_{cv}}{T_o} - \sum_i (\dot{m}s)_e + \sum_e (\dot{m}s)_e + \dot{\sigma}_{surr}$ → 0

- $\frac{dS_{net}}{dt} = \frac{dS_{cv}}{dt} + \frac{dS_{surr}}{dt} = \frac{\dot{Q}_{cv}}{T_i} - \frac{\dot{Q}_{cv}}{T_o} + \dot{\sigma}_{tot}$
- $\dot{\sigma}_{tot} \geqslant 0$, $\longrightarrow$ $\boxed{\frac{dS_{net}}{dt} = \frac{dS_{cv}}{dt} + \frac{dS_{surr}}{dt} \geqslant 0}$

Borgnakke Ex. 7.2: ▷ Reversible adiabatic flow of steam through nozzle, $V_e = ?$



T185

- Continuity equation: $\dot{m}_i = \dot{m}_e = \dot{m}$
- First law: $0 = 0 - 0 + (h_i - h_e) + \frac{V_i^2 - V_e^2}{2} + 0$
- Second law: $s_i = s_e$.

$\Rightarrow$ $h_i = h(steam,\ P_i = 1\ MPa,\ T_i = 300^oC) = \checkmark$

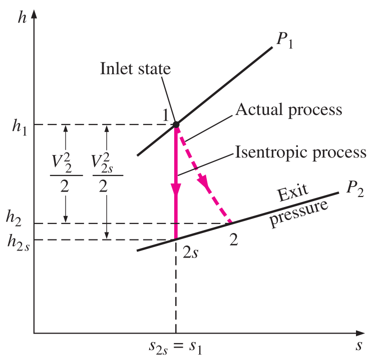
$\Rightarrow$ $s_i = s(steam,\ P_i = 1\ MPa,\ T_i = 300^oC) = \checkmark$

$\Rightarrow$ $s_e = s_i$ & $P_e = 0.3$ MPa: state 'e' defined.

$\Rightarrow$ $h_e = h(P_e = 0.3\ MPa,\ s_e = \checkmark) = \checkmark$

$\Rightarrow$ $\frac{V_e^2}{2} = (h_i - h_e) + \frac{V_i^2}{2} \Rightarrow V_e = 736.7$ m/s◁

Isentropic Nozzle Efficiency

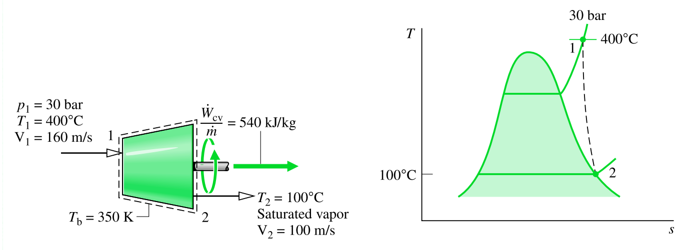


T181

- For nozzle: $w = 0$, $\Delta(PE) = 0$, $V_1 \sim 0 \Rightarrow h_1 = h_2 + \frac{V_2^2}{2}$

$\Rightarrow$ $\boxed{\eta_N = \frac{\text{Actual KE at nozzle exit}}{\text{Isentropic KE at nozzle exit}} = \frac{V_2^2}{V_{2s}^2} \approx \frac{h_1 - h_2}{h_1 - h_{2s}}}$

Moran Ex. 6.6: ▷ Entropy production in a steam turbine



T178

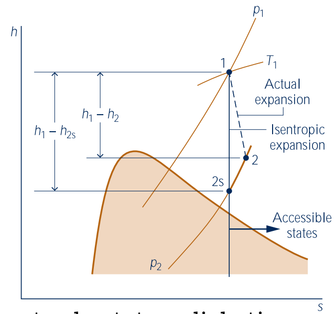
- Continuity equation: $\dot{m}_i = \dot{m}_e = \dot{m}$
- First law: $0 = q - w + (h_i - h_e) + \frac{V_i^2 - V_e^2}{2} + 0$
- Second law: $(s_e - s_i) = \frac{\dot{q}}{T_b} + \frac{\dot{\sigma}_{cv}}{\dot{m}}$.

⇒ $P_1 = 30$ bar, $T_1 = 400^oC \Rrightarrow h_1 = \checkmark$, $s_1 = \checkmark$

⇒ $T_2 = 100^oC$, $x_2 = 1.0 \Rrightarrow h_2 = \checkmark$, $s_2 = \checkmark$

⇒ $q = -23.92$ kJ/kg $\Rightarrow \frac{\dot{\sigma}_{cv}}{\dot{m}} = 0.499$ kJ/kg.K ◁.

Isentropic Turbine Efficiency

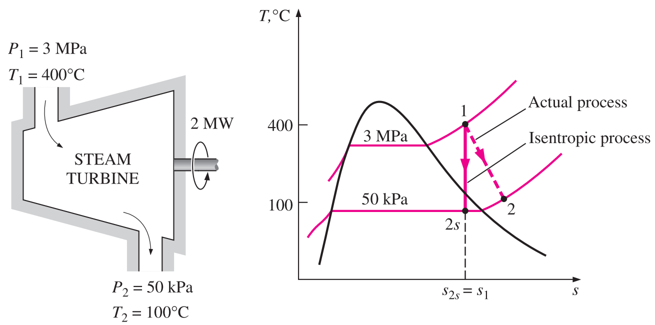


T176

- $\dot{w}_t = h_1 - h_2$: for steady-state, adiabatic expansion.
- $\frac{\dot{\sigma}_{cv}}{\dot{m}} = s_2 - s_1 \geqq 0$: states with $s_2 < s_1$ is not attainable with adiabatic expansion.
- $\dot{w}_t|_s = h_1 - h_{2s}$: state '2s' is for internally reversible expansion.

⇒ Isentropic turbine efficiency, $\boxed{\eta_t = \frac{\dot{w}_t}{\dot{w}_t|_s} = \frac{h_1 - h_2}{h_1 - h_{2s}}}$

Cengel Ex.7.14: ▷ Isentropic Efficiency of a Steam Turbine



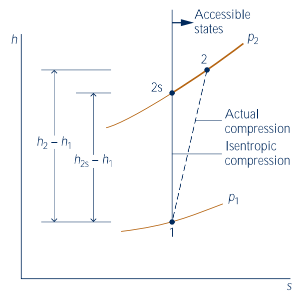
T179

- $(P_1 = 3$ MPa, $T_1 = 400^oC) :\Rrightarrow h_1 = \checkmark, s_1 = \checkmark$
- $(P_2 = 50$ kPa, $T_2 = 100^oC) :\Rrightarrow h_2 = \checkmark$
- $(P_{2s} = 50$ kPa, $s_{2s} = s_1) :\Rrightarrow h_{2s} = \checkmark$

⇒ $\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} = 66.7\%$ ◁

⇒ $\dot{m} = \frac{Power}{h_1 - h_{2s}} = 3.64$ kg/s ◁

Isentropic Compressor and Pump Efficiencies

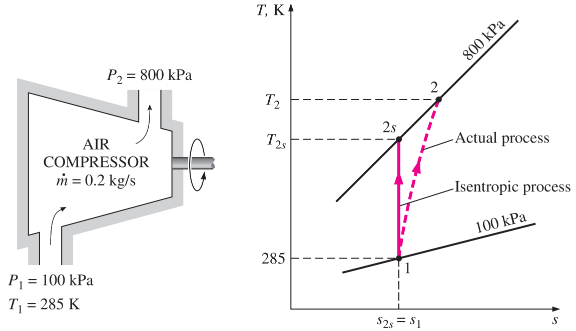


T177

- $\dot{w}_c = h_1 - h_2 = -(h_2 - h_1)$: for steady-state, adiabatic compression.
- $\frac{\dot{\sigma}_{cv}}{\dot{m}} = s_2 - s_1 \geqq 0$: states with $s_2 < s_1$ is not attainable with adiabatic compression.
- $\dot{w}_c|_s = h_1 - h_{2s} = -(h_{2s} - h_1)$

⇒ Isentropic compressor/pump efficiency, $\boxed{\eta_c = \frac{-\dot{w}_c|_s}{-\dot{w}_c} = \frac{h_{2s} - h_1}{h_2 - h_1}}$

Cengel Ex. 7.15: ▷ Effect of efficiency on compressor power, if $\eta_c = 80\%$



T180

- $T_{2s} = T_1(P_2/P_1)^{(k-1)/k} = 516.5$ K
- $\eta_c = \frac{h_{2s}-h_1}{h_2-h_1} = \frac{c_P(T_{2s}-T_1)}{c_P(T_2-T_1)} \Rightarrow T_2 = 574.8$ K

$\Rightarrow$ $W_c|_s = -\dot{m}(h_{2s} - h_1) = \dot{m}c_P(T_{2s} - T_1) = -46.5$ kW ◁

$\Rightarrow$ $W_c = -\dot{m}(h_2 - h_1) = -\dot{m}c_P(T_2 - T_1) = -58.1$ kW ◁

Steady-state Flow Process

- First Law: CM system & reversible process:

$\Rightarrow$ $\delta q = du + \delta w = d(h - Pv) + Pdv = dh - vdP$

- Second Law: $\delta q = Tds \Rightarrow Tds = dh - vdP$

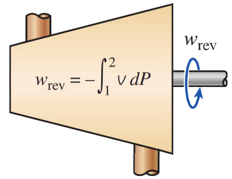
$\Rightarrow$ $\int_1^2 Tds = \int_1^2 (dh - vdP) = (h_2 - h_1) - \int_1^2 vdP$

- First Law: CV system & reversible process:

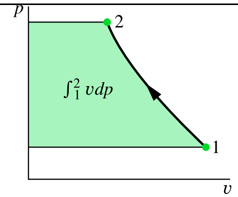
$\Rightarrow$ $0 = q_{12} - w_{12} + (h_1 - h_2) - \Delta(ke) - \Delta(pe)$

$\Rightarrow$ $w_{12} = q_{12} + (h_1 - h_2) - 0 - 0 = \int_1^2 Tds + (h_1 - h_2) = -\int_1^2 vdP$

$$w_{sf} = w_{12} = -\int_1^2 vdP$$



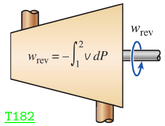
T182



T183

$$Pv^n = \text{constant}$$

$$w_{sf} = w_{12} = -\int_1^2 vdP$$



T182

$$w_{12} : \begin{cases} = \frac{n}{1-n}(P_2v_2 - P_1v_1) & = \frac{nR(T_2-T_1)}{1-n} & : n \neq 1 \\ = RT\ln\left(\frac{v_2}{v_1}\right) & = -RT\ln\left(\frac{P_2}{P_1}\right) & : n = 1 \end{cases}$$

$$q_{12} = h_2 - h_1 + w_{12}$$

Example: ▷ A compressor operates at steady state with nitrogen entering at 100 kPa and 20°C and leaving at 500 kPa. During this compression process, the relation between pressure and volume is $Pv^{1.3}$ =constant.

- $R = (8.314/28) = 0.2969$ kJ/kg.K, $c_P = 1.0$ kJ/kg.K

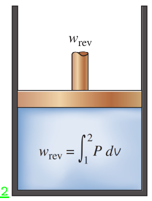
$\Rightarrow$ $T_2/T_1 = (P_2/P_1)^{(n-1)/n} \Rightarrow T_2 = 425$ K.

$\Rightarrow$ $w_{12} = \frac{nR(T_2-T_1)}{1-n} = -169.5$ kJ/kg ◁

$\Rightarrow$ $q_{12} = c_P(T_2 - T_1) + w_{12} = -37.5$ kJ/kg ◁

$$Pv^n = \text{constant}$$

$$w_b = w_{12} = \int_1^2 Pdv$$



T192

$$w_{12} : \begin{cases} = \frac{P_2v_2 - P_1v_1}{1-n} & = \frac{R(T_2-T_1)}{1-n} & : n \neq 1 \\ = RT\ln\left(\frac{v_2}{v_1}\right) & = -RT\ln\left(\frac{P_2}{P_1}\right) & : n = 1 \end{cases}$$

$$q_{12} = u_2 - u_1 + w_{12}$$

Example: ▷ In a reversible process, nitrogen is compressed in a cylinder from 100 kPa and 20°C to 500 kPa. During this compression process, the relation between pressure and volume is $Pv^{1.3}$ =constant.

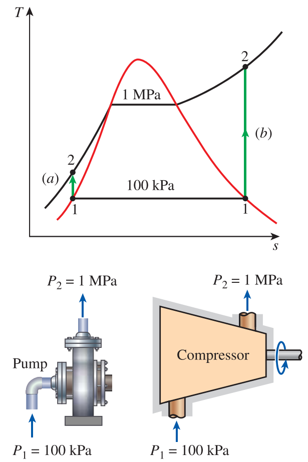
- $R = (8.314/28) = 0.2969$ kJ/kg.K, $c_v = R/(1.4 - 1) = 0.742$ kJ/kg.K

$\Rightarrow$ $T_2/T_1 = (P_2/P_1)^{(n-1)/n} \Rightarrow T_2 = 425$ K.

$\Rightarrow$ $w_{12} = \frac{R(T_2-T_1)}{1-n} = -130.4$ kJ/kg ◁

$\Rightarrow$ $q_{12} = c_V(T_2 - T_1) + w_{12} = -32.2$ kJ/kg ◁

Cengel Ex.7.12: ▷ Compression & Pumping Works



(a) Compressing a liquid
(b) Compressing a vapor

T191

(a) $w_P = -\int_1^2 v dP \simeq -v_f(P_2 - P_1) = -0.939$ kJ/kg

(b) $w_C = h_1 - h_2 = -518.6$ kJ/kg

$\Rightarrow$ $w_c >> w_p$