Properties of Homogeneous Mixtures & Pyschrometry

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ME 203: Engineering Thermodynamics

Ideal Gas Mixtures

- $m = m_1 + m_2 + m_3 + \cdots + m_k = \sum_{i=1}^{k} m_i$

$$mf_i \equiv \frac{m_i}{m} \quad :\Rightarrow \quad \sum_{i=1}^{k} mf_i = 1$$

- $n = n_1 + n_2 + n_3 + \cdots + n_k = \sum_{i=1}^{k} n_i$

$$Y_i \equiv \frac{n_i}{n} \quad :\Rightarrow \quad \sum_{i=1}^{k} Y_i = 1$$

m_i = mass of component i
m = total mass of mixture
n_i = number of moles of component i
n = total number of moles in mixture
mf_i = mass fraction of component i
Y_i = mole fraction of component i
M_i = molecular mass of component i
M = apparent molecular mass of mixture i

$$m_i = n_i M_i \quad : \quad m = nM$$

- $M = \frac{m}{n} = \frac{m_1 + m_2 + \cdots + m_k}{n} = \frac{n_1 M_1 + n_2 M_2 + \cdots + n_k M_k}{n} = \sum_{i=1}^{k} \left(\frac{n_i}{n}\right) M_i$

$$M = \sum_{i=1}^{k} Y_i M_i$$

  - ▶ Dry air (Mole basis): $78.08\% N_2, 20.95\% O_2, 0.93\% Ar, 0.03\% CO_2$:

    $M = 0.7808(28.02) + 0.2095(32) + 0.0093(39.94) + 0.0003(44) = 28.96$ kg/kmol

- Apparent gas constant, $R = \frac{R_u}{M}$

  - ▶ Dry air: $R = \frac{8.314}{28.96} = 0.287$ kJ/kg K

- $M = \frac{m}{n} = \frac{m}{\sum_{i=1}^{k} n_i} = \frac{m}{\sum_{i=1}^{k} \left(\frac{m_i}{M_i}\right)} = \frac{1}{\sum_{i=1}^{k} \left(\frac{m_i/m}{M_i}\right)} = \frac{1}{\sum_{i=1}^{k} \left(\frac{mf_i}{M_i}\right)}$

$$M = \frac{1}{\sum_{i=1}^{k} \left(\frac{mf_i}{M_i}\right)}$$

- $mf_i = \frac{m_i}{m} = \frac{n_i M_i}{\sum n_i M_i} = \frac{n_i M_i / n}{\sum n_i M_i / n} = \frac{Y_i M_i}{\sum Y_i M_i} \quad :\Rightarrow \quad mf_i = \frac{Y_i M_i}{M}$

$$mf_i = \frac{Y_i M_i}{M} = \frac{Y_i M_i}{\sum Y_i M_i} \quad : \quad Y_i = \left(\frac{mf_i}{M_i}\right) M = \frac{mf_i / M_i}{\sum mf_i / M_i}$$

Conversion: Mass Fraction ⇌ Mole Fraction

- $M = \sum_{i=1}^{k} Y_i M_i$
- $Y_i = \left(\frac{mf_i}{M_i}\right) M = \frac{mf_i / M_i}{\sum mf_i / M_i}$

i	mf_i	M_i	$\frac{mf_i}{M_i}$	Y_i
H_2	0.10	2.0	0.050	0.6250
O_2	0.48	32.0	0.015	0.1875
CO	0.42	28.0	0.015	0.1875
	1.00	-	0.080	1.0000

$M = \frac{1}{\sum mf_i / M_i} = \frac{1}{0.080} = 12.5$ kg/kmol.

- $M = \frac{1}{\sum_{i=1}^{k} \left(\frac{mf_i}{M_i}\right)}$
- $mf_i = \frac{Y_i M_i}{M} = \frac{Y_i M_i}{\sum Y_i M_i}$

i	Y_i	M_i	$Y_i M_i$	mf_i
H_2	0.6250	2.0	1.25	0.10
O_2	0.1875	32.0	6.00	0.48
CO	0.1875	28.0	5.25	0.42
	1.0000	-	12.5	1.00

$M = \sum Y_i M_i = 12.5$ kg/kmol.

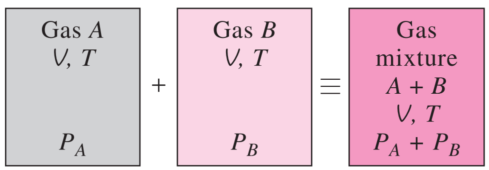
P-v-T Behaviour of Gas Mixtures

Dalton's Law of Additive Pressures

The pressure of a gas mixture is equal to the sum of the pressures each gas would exert if it existed alone at the mixture temperature and volume.

$$P = P_1 + P_2 + \cdots + P_k = \sum_{i=1}^{k} P_i(T, V)$$

P_i = partial pressure of component i.



T281

$$P = P_A + P_B$$

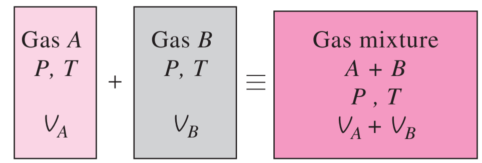
Amagat's Law of Additive Volumes

The volume of a gas mixture is equal to the sum of the volumes each gas would occupy if it existed alone at the mixture temperature and pressure.

$$V = V_1 + V_2 + \cdots + V_k = \sum_{i=1}^{k} V_i(T, P)$$

V_i = component volume of component i.

T282



$$V = V_A + V_B$$

$$\Longrightarrow \frac{P_i(T,V)}{P} = \frac{n_i R_u T/V}{n R_u T/V} = \frac{n_i}{n} = Y_i \;:\; \frac{V_i(T,P)}{V} = \frac{n_i R_u T/P}{n R_u T/P} = \frac{n_i}{n} = Y_i$$

$$Y_i \equiv \frac{n_i}{n} = \frac{P_i}{P} = \frac{V_i}{V} \qquad \text{(Ideal gas)}$$

Gibbs-Dalton's Law

In a mixture of ideal gases each component of the mixture acts as if it were alone in the system at the volume V and the temperature T of the mixture.

- $U = U_1 + U_2 + \cdots + U_k = \sum_{i=1}^{k} U_i$
- $U = n\tilde{u} = n_1\tilde{u}_1 + n_2\tilde{u}_2 + \cdots + n_k\tilde{u}_k = \sum_{i=1}^{k} n_i\tilde{u}_i$

  $\tilde{u} \equiv$ specific internal energy on mole basis.

$\Longrightarrow$ $\tilde{u} = \frac{U}{n} = Y_1\tilde{u}_1 + Y_2\tilde{u}_2 + \cdots + Y_k\tilde{u}_k = \sum_{i=1}^{k} Y_i\tilde{u}_i$

- $U = mu = m_1u_1 + m_2u_2 + \cdots + m_ku_k = \sum_{i=1}^{k} m_iu_i$

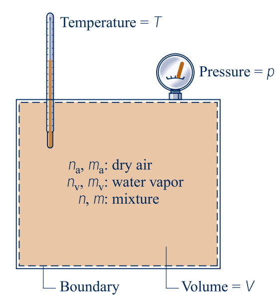
$\Longrightarrow$ $u = \frac{U}{m} = mf_1u_1 + mf_2u_2 + \cdots + mf_ku_k = \sum_{i=1}^{k} mf_iu_i$

  $u \equiv$ specific internal energy on mass basis.

- $\tilde{u} = \sum Y_i\tilde{u}_i \;:\; \tilde{h} = \sum Y_i\tilde{h}_i \;:\; \tilde{s} = \sum Y_i\tilde{s}_i \;:\; \tilde{g} = \sum Y_i\tilde{g}_i$
- $u = \sum mf_iu_i \;:\; h = \sum mf_ih_i \;:\; s = \sum mf_is_i \;:\; g = \sum mf_ig_i$

Moist Air

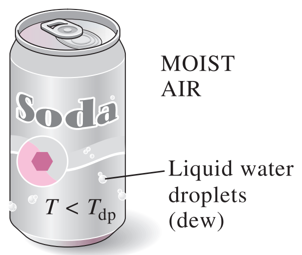
- Atmospheric air contains several gaseous components including water vapour and contaminants such as dust and pollutants.
- Dry air refers only to the gaseous components when all water vapour and contaminants have been removed.
- Moist air refers to a mixture of dry air and water vapour in which the dry air is treated as if it were a pure component.



T284

- $P = P_a + P_v$
- $n = n_a + n_v$
- $m = m_a + m_v$
- $Y_a = \frac{n_a}{n} \;:\; Y_v = \frac{n_v}{n}$
- $P_a = Y_aP \;:\; P_v = Y_vP$

Dew-point Temperature, T_{dp}



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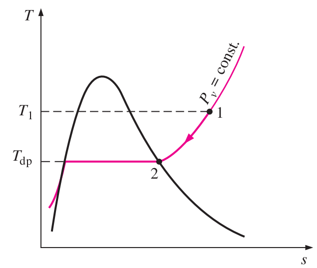


T287

When the temperature of a cold drink is below the T_{dp} of the surrounding air, it sweats.

Constant-pressure cooling of moist air and the dew-point temperature on the T-s diagram of water.

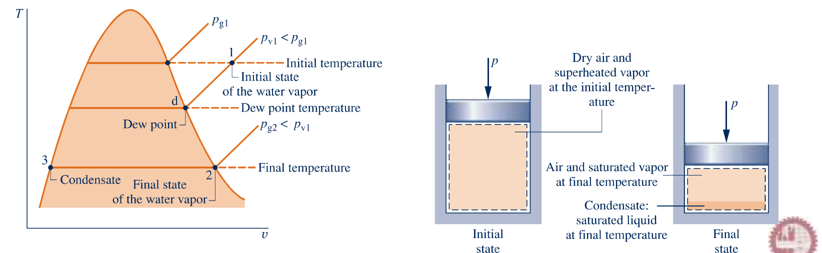
$$T_{dp} = T_{sat}(P_v)$$

Relative Humidity, ϕ and Moisture Content, ω

- Relative humidity, $\phi \equiv \left.\frac{P_v}{P_g}\right]_{T,P}$: $P_g = P_{sat}(T)$
- Moisture content, $\omega \equiv \frac{m_v}{m_a}$

→ $\omega = \frac{m_v}{m_a} = \frac{P_v V M_v / R_u T}{P_a V M_a / R_u T} = \frac{M_v}{M_a}\frac{P_v}{P_a} \simeq 0.622 \frac{P_v}{P_a}$: $18.0/28.95 = 0.622$

$$\omega = 0.622 \frac{P_v}{P_a} = 0.622 \frac{P_v}{P - P_v} = 0.622 \frac{\phi P_g}{P - \phi P_g}$$



T285

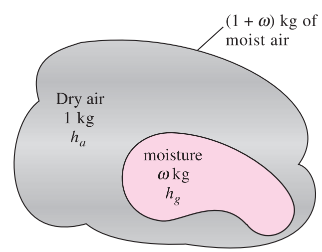
▷ [Borgnakke 11.2]: Consider 100 m^3 of an air-water vapour mixture at 0.1 MPa, 35°C, and 70% relative humidity. Calculate the humidity ratio, dew point, mass of air, and mass of vapour.

- $P_v = \phi P_g = 0.70(5.628) = 3.94$ kPa $\mapsto T_{dp} = T_{sat}(P_v) = 28.6$°C ◀
- $P_a = P - P_v = 100 - 3.94 = 96.06$ kPa $\mapsto \omega = 0.0255$ kg/kg da ◀
- $m_a = P_a V / R_a T = 96.06 \times 100)/(0.287 \times 308.2) = 108.6$ kg ◀
- $m_v = \omega m_a = 0.0255(108.6) = 2.77$ kg ◀

▷ [Borgnakke 11.3]: Calculate the amount of water vapour condensed if the mixture of Example 11.2 is cooled to 5°C in a constant-pressure process.

- $T_2 < T_{dp}$: so condensation occurs and $\phi_2 = 1.0$
- $P_{v2} = P_g(T_2) = 0.8721$ kPa $\mapsto P_{a2} = 100 - 0.8721 = 99.128$ kPa
- $\omega_2 = 0.622 \times 0.8721/99.128 = 0.0055$ kg/ kg da; $\omega_1 = 0.0255$ kg/kg da
- $m_{w2} \equiv$ water condensation, $m_{v1} = m_{v2} + m_{w2}$
- $\mapsto \frac{m_{w2}}{m_a} = \frac{m_{v1} - m_{v2}}{ma} \mapsto \boxed{m_{w2} = m_a(\omega_1 - \omega_2)} = 2.172$ kg◀
- If heated to 35°C, $\phi = 15\%$ (very dry)

Moist Air Enthalpy, h

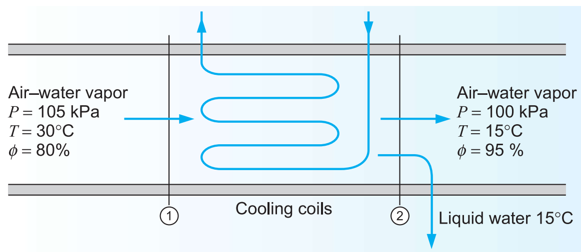


T283

$h = h_a + \omega h_g$, kJ/kg dry air

- $H = H_a + H_v = m_a h_a + m_v H_v$
- $h \equiv \frac{H}{m_a} = h_a + \frac{m_v}{m_a} h_v = h_a + \omega h_v$
- $h_v \simeq h_g(T)$ $:\Longrightarrow$ $\boxed{h = h_a + \omega h_g}$
- $h_a = c_{pa} T = 1.005 T$ [kJ/kg da]
- $h_w = c_{pw} T = 4.1867 T$ [kJ/kg water]
- $h_g = 2501.3 + 1.82 T$ [kJ/kg water vapour] ⇐ T in °C.

▷ [Borgnakke 11.5]: Cooling and dehumidification in a cooling coil of an AC:

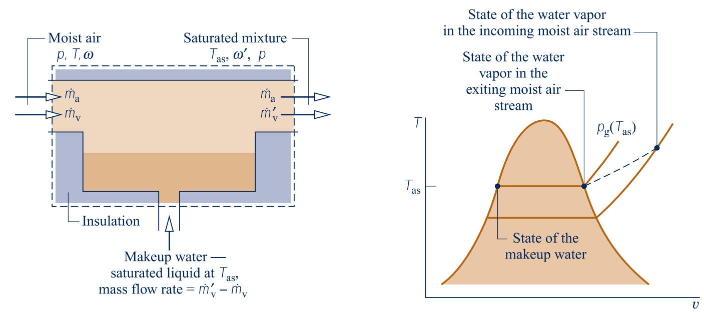


T295

- Mass balance: $\dot{m}_{a1} = \dot{m}_{a2} = \dot{m}_a \; : \; \dot{m}_{v1} = \dot{m}_{v2} + \dot{m}_{w2}$
- Energy balance: $\dot{Q}_{cv} + \sum \dot{m}_i h_i = \sum \dot{m}_e h_e$
- $\phi = \frac{P_v}{P_g} \; : \; \omega = 0.622 \frac{\phi P_g}{P - \phi P_g}$
- $h_a = 1.005\,T \; : \; h_v = h_g = 2501.3 + 1.82\,T \; : \; h_w = 4.186\,T \; : \; h = h_a + \omega h_v$

$$\frac{\dot{Q}_{cv}}{\dot{m}_a} = (h_{a2} + \omega_2 h_{v2}) - (h_{a1} + \omega_1 h_{v1}) + (\omega_1 - \omega_2) h_{w2}$$
$$= (h_2 - h_1) + (\omega_1 - \omega_2) h_{w2} = -41.64 \text{ kJ/kg da} ◀$$

Adiabatic Saturation Process

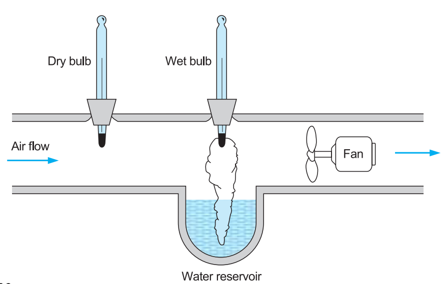


T288

- $h_v(T) \simeq h_g(T),\; h_v(T_{as}) \simeq h_g(T_{as}),\; \omega = m_v/m_a,\; \omega' = m'_v/m_a$
- $m_a h_a(T) + m_v h_g(T) + (m'_v - m_v) h_w(T_{as}) =$
$m_a h_a(T_{as}) + m'_v h_g(T_{as})$

$$\omega = \frac{h_a(T_{as}) - h_a(T) + \omega'[h_g(T_{as}) - h_f(T_{as})]}{h_g(T) - h_f(T_{as})} \; : \; \omega' = 0.622 \frac{p_g(T_{as})}{P - P_g(T_{as})}$$

Wet Bulb Temperature, T_{wb} and Psychrometer



T309

Adiabatic saturation process provides a mean to measure humidity content of moist air, and the process can be approximated by using a wet-bulb thermometer.

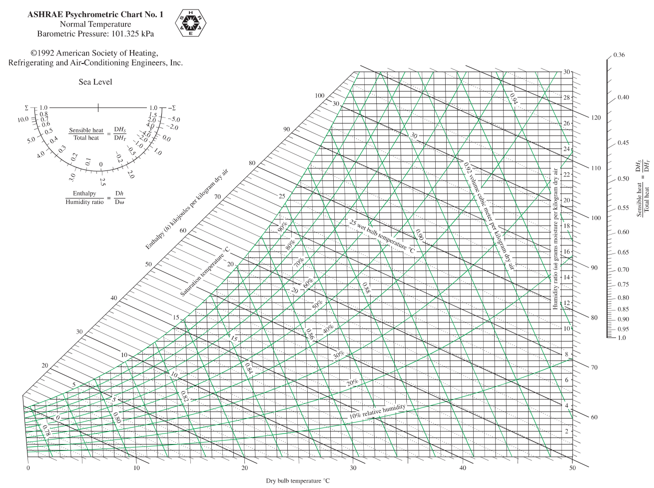
- Wet-bulb temperature, T_{wb} is read from a wet-bulb thermometer, which is an ordinary liquid-in-glass thermometer whose bulb is enclosed by a wick moistened with water.
- Dry-bulb temperature, T_{db} refers simply to the temperature that would be measured by a thermometer placed in the mixture. Often a wet-bulb thermometer is mounted together with a dry-bulb thermometer to form an instrument called a psychrometer.

▷ Example: $T_{db} = 25^o C$ & $T_{wb} = 20^o C$:

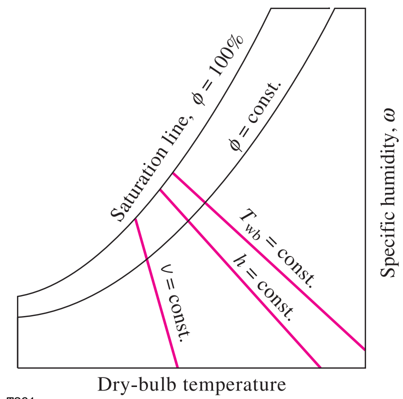
- $P_g(T) = P_{sat}(T) :\Rightarrow P_g(T_{db}) = 3.169$ kPa, $P_g(T_{wb}) = 2.339$ kPa
- $\phi' = 1.0, \quad \omega' = 0.622 \frac{\phi_2 P_g(T_{wb})}{P - \phi_2 P_g(T_{wb})} \Rightarrow \omega' = 0.0147$
- $h_a(T_{wb}) - h_a(T_{db}) = 1.005(T_{wb} - T_{db}) \; : \; h_w(T_{wb}) = 4.186\,T_{wb}$
- $h_g(T_{db}) = 2547.2$ kJ/kg, $h_g(T_{wb}) = 2538.1$ kJ/kg
- $\omega = \frac{h_a(T_{wb}) - h_a(T_{db}) + \omega'[h_g(T_{wb}) - h_w(T_{wb})]}{h_g(T_{db}) - h_f(T_{wb})} = 0.0126$ ◀
- $\omega = 0.622 \frac{\phi P_g(T_{db})}{P - \phi P_g(T_{db})} \Rightarrow \phi = 0.635$ ◀
- $h = h_a(T_d b) + \omega h_g(T_{db}) = 57.23$ kJ/kgda ◀
- $P_v = \phi P_g(T_{db}) = 2012.5$ kPa
- $T_{dp} = T_{sat}(P_v) = 17.6^o C$ ◀

Note That: By default, $P = 101.325$ kPa. Psychrometric charts are prepared for atmospheric pressure.

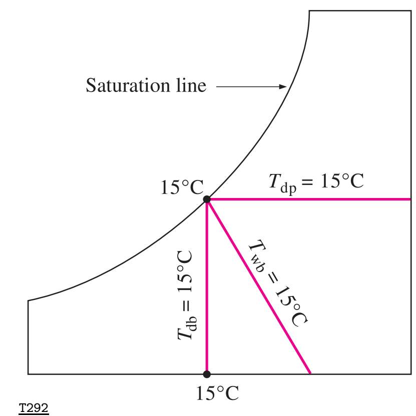
Psychrometric Chart



T290

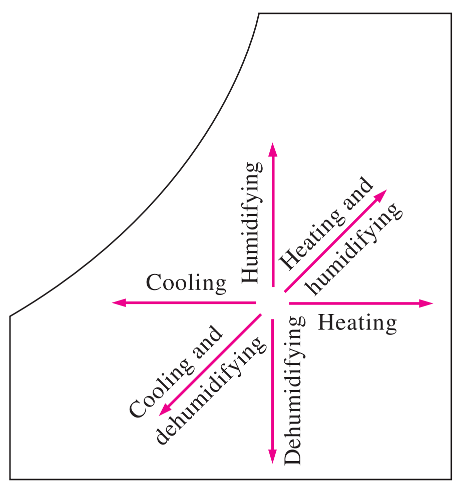


T291



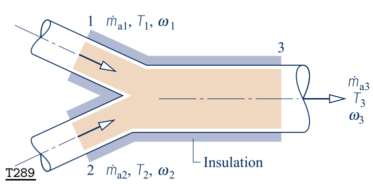
T292

Psychrometric Processes

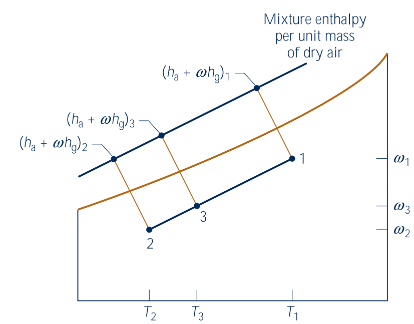


T294

Adiabatic Mixing of Two Moist Air Streams



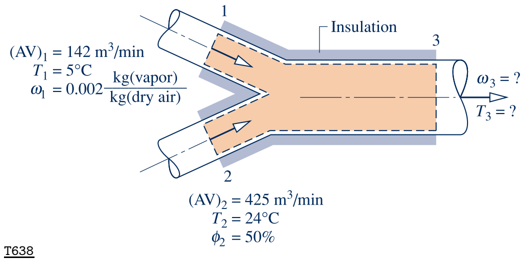
T289



- Dry air : $\dot{m}_{a1} + \dot{m}_{a2} = \dot{m}_{a3}$
- Water vapour : $\dot{m}_{v1} + \dot{m}_{v2} = \dot{m}_{v3} \implies \boxed{\omega_3 = \frac{\omega_1 \dot{m}_{a1} + \omega_2 \dot{m}_{a2}}{\dot{m}_{a1} + \dot{m}_{a2}}}$
- $\dot{m}_{a1}(h_{a1} + \omega_1 h_{v1}) + \dot{m}_{a2}(h_{a2} + \omega_2 h_{v2}) = \dot{m}_{a3}(h_{a3} + \omega_3 h_{v3})$

$\implies \dot{m}_{a1} h_1 + \dot{m}_{a2} h_2 = \dot{m}_{a3} h_3 \implies \boxed{h_3 = \frac{h_1 \dot{m}_{a1} + h_2 \dot{m}_{a2}}{\dot{m}_{a1} + \dot{m}_{a2}}}$

▷ [Moran 12.14]: Using psychrometric chart, solve the problem:



T638

- Psychrometric chart:
- $v_{a1} = 0.79$ m^3 /kg
- $v_{a2} = 0.855$ m^3 /kg
- $\omega_2 = 0.0094$ kg/kg da

- $m_{a1} = \rho_{a1}(AV)_1 = \frac{(AV)_1}{v_{a1}} = 3.0$ kg/s
- $m_{a2} = \rho_{a2}(AV)_2 = \frac{(AV)_2}{v_{a2}} = 6.06$ kg/s
- $\omega_3 = \frac{\omega_1 \dot{m}_{a1} + \omega_2 \dot{m}_{a2}}{\dot{m}_{a1} + \dot{m}_{a2}} = 0.0094$ kg/kg da ◀

⇒ Psychometric chart: state ③ will be on the line joining ① and ②.

- $T_3 = 19$°C ◀

▷ Air at 0.1 MPa, 30°C and 80% RH is compressed isothermally to 1.0 MPa. Estimate the psychrometric condition of the compressed air.

- ①: $P_1 = 0.1$MPa, $\phi = 0.8 \rightsquigarrow \omega_1 = 0.0217$ kg/kg da
- ②:If $\omega_1 = \omega_2 = 0.622 \frac{\phi_d P_{v2}}{P - \phi_2 p_{v2}} \rightsquigarrow \phi_2 = 7.88 > 1.0$
- So water condensation occurs, and $\phi_2 = 100\%$
- $\omega_2 = 0.622 \frac{\phi_d P_{v2}}{P - \phi_2 p_{v2}} = 0.622 \frac{0.00425}{1.0 - 0.00425} = 0.00265$ kg/kg da
- $\Delta w = (\omega_1 - \omega_2) = 0.019$ kg/kg da (condensation)